Dynamo or Gyroscope?

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1 Abstract

Generation of cosmic magnetic fields is presently understood as happening by self-inductive action in liquid, electrically conducting and rotating bodies. This study proposes a different explanation of planetary and astrophysical magnetism. The mechanism is based on collective azimuthal motion of conducting electrons as a result of balance of the classical electron gas in a rotating body. It relies on weak magnetic properties of the concerned material and on the effect of magnetization by rotation known as Barnett's effect. The behaviour of electrons is likened to gyroscopic motion to which angular momentum and therefore also magnetic moment is attributed. The problem is governed by the law of conservation of angular momentum and is examined following Euler analysis of the rigid-body rotation. The presented conception offers a simple interpretation of cosmic magnetic fields themselves, variety of their configurations as well as of some their observed changes including magnetic reversals.

Keywords: Cosmic magnetism; Electron gas; Angular momentum; Magnetic moment

2 Introduction

Cosmic magnetic fields, which are accessible for observations, show substantial variety of geometries and magnitudes. The dipolar character prevails but the dipole tilt and offset with respect to the rotational axis differs from case to case. These large-scale fields cannot come from permanent magnetism due to high temperatures of interiors of cosmic bodies exceeding the Curie point. It is therefore believed that they are generated in electrically conducting parts of cosmic bodies by equal principle and the dynamo theory provides a feasible explanation of their existence. The hydromagnetic dynamo theory is based on primary ideas by Joseph Larmor in 1919 who attempted to explain the origin of Sun's magnetic field. He suggested that motion of an electrically conducting fluid across magnetic field within a rotating body might by its inductive action give rise to such currents that regenerate the original magnetic field. Since that time, amounts of analytical and later also numerical and experimental studies were performed in order to understand principal and finer features of this complex problem. The fundamentals of the dynamo theory are well presented in [1]. Today it is believed that generation of magnetic field via a self-sustaining dynamo has three basic requirements on a cosmic body. First, it must possess a large volume of electrically conducting (non-magnetic) fluid. Second, the fluid must be in motion, so an energy source for convection is necessary. Third, rotation is essential in order to organize the fluid motion in a way to be convenient for the regeneration process. The whole process is mathematically described by the induction equation for the magnetic field $\mathbf{B}$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{1}$$

together with the equation of motion for the fluid velocity $\mathbf{u}$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\mathbf{u} + 2\,\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \frac{1}{\rho}\mathbf{j} \times \mathbf{B} + \mathbf{G} + \nu \nabla^2 \mathbf{u} \tag{2}$$

completed by the equation of heat (or light elements) transport. Here t is time in which the evolution takes place, $\boldsymbol{\Omega}$ is angular velocity of the fluid, p is pressure, $\mathbf{G}$ stands for a body force per unit mass, ρ is density of the fluid and $\mathbf{j}$ is electric current density. The parameters η and ν are magnetic diffusivity and fluid viscosity, respectively. Although generally accepted, there are still many open problems the dynamo theory is not able to resolve. Some of them are listed in [2]. Besides of this, there seem to be several inconsistencies in the theory, which incite to fundamental issues.

2.1 Electric currents in a bulk conductor

What is the origin of the current $\mathbf{j}$ in the Ampère's force term in (2)? If the convective current $\rho_e \mathbf{u}$, where ρ_e is electric charge density, is not considered, $\mathbf{j}$ must result from presence of some electric field $\mathbf{E}$. There are two types of electric intensities responsible for currents in a conductor and entering the Ohm's law

$$\mathbf{j} = \sigma \mathbf{E}. \tag{3}$$

The first one, $\mathbf{E}_{st}$, results from distribution of (uncompensated) electric charge. This is the electrostatic field whose essential property is that

$$\oint_l \mathbf{E}_{st} \cdot \mathrm{d}\mathbf{l} = \int_S (\boldsymbol{\nabla} \times \mathbf{E}_{st}) \cdot \mathrm{d}\mathbf{S} = 0. \tag{4}$$

It is an irrotational field whose field lines are not closed and with the aid of a scalar potential U it can be expressed as

$$\mathbf{E}_{st} = -\boldsymbol{\nabla} U. \tag{5}$$

The field $\mathbf{E}_{st}$ is always zero inside a conductor unless it is held on some potential difference. This is not the case. Then there is the induced electric field which arises from movement of the conductor in magnetic field $\mathbf{B}$ or from changeability of the magnetic field in time. These two effects transmute into each other in the moving frame causing a change of magnetic flux $\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}$ through the surface of the conductor

$$\oint_l \mathbf{E}_{ind} \cdot \mathrm{d}\mathbf{l} = \oint_l (\mathbf{u} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} = \int_S \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{S} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}. \tag{6}$$

Provided the surface S remains constant, it can be written

$$\boldsymbol{\nabla} \times \mathbf{E}_{ind} = -\frac{\partial \mathbf{B}}{\partial t} \tag{7}$$

which is the Faraday's law of induction. According to it, motion of a bulk conductor in magnetic field gives rise to eddy currents, which act against this relative change and brake the motion. It is impossible for the flow to organize itself and to design any special paths for currents in order to break the Lenz's Law (represented by the minus sign in (7)) and give a generational effect the dynamo theory requires. Eddy currents always form a pair of closed loops producing self magnetic field with such configuration as to compensate the change in magnetic flux through the conductor. They last as long as this change persists and rapidly dissipate. On principle, they cannot sustain the primary magnetic field. Equally, the secondary magnetic field associated with them cannot be identified with the original one like it is accomplished through the Ampère's law in (2) when a dynamo is being modelled. The Faraday's law fully defines the process of electromagnetic induction.

But the dynamo theory insists that both inductive effects coexist giving

$$\boldsymbol{\nabla} \times \mathbf{E}_{ind} = -\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}). \tag{8}$$

Utilizing the Ampère's law

$$\mathbf{j} = \frac{1}{\mu_0} \boldsymbol{\nabla} \times \mathbf{B}, \tag{9}$$

where μ_0 is the magnetic permeability of vacuum, (8) then leads to the induction equation (1) which should describe evolution of the magnetic field $\mathbf{B}$ as a competition between its production and diffusion. Actually, the term $\partial \mathbf{B}/\partial t$ is misunderstood to represent evolution of the field $\mathbf{B}$ whereas it is only its time change as it appears to the conductor at a particular point. Eventual evolution of global $\mathbf{B}$, if it really takes place, is absolutely independent of the mentioned inductive effects. Another inconsistency lies in the usage of (9). The Ampère's law expresses vorticity of $\mathbf{B}$ as a local property of magnetic field; everywhere electric currents flow, the magnetic field creates vortices. In other words, the magnetic field appearing in (9) is not the same as the field in (8), but only a response to electromagnetic induction due to change of the global magnetic field. As it was said above, its character is totally different than the one of the original field implying that the induction equation is just a mathematical construct which does not reflect the physical reality.

Another fundamental question suggests itself: where the original magnetic field comes from? A dynamo, which is considered to be workable, always needs a primary seed field to later evolve in symbiosis with the flows of the fluid. The dynamo theory does not explain origination of a particular magnetic field. It only relies on existence of omnipresent galactic magnetic field which should serve as a seed field for astrophysical dynamos. Before the dynamo theory started to develop, there was a general belief amongst physicists that mechanical analogies can help in understanding electromagnetic phenomena. Magnetization was supposed to be a state of matter where spinning particles acting as gyrostats were oriented in a common direction. Rotation should therefore result in magnetism in an appropriate material. This idea inspired Barnett who speculated on the origin of terrestrial magnetism. In [3], he suggested that magnetized matter is just an oriented atomic or molecular system with individual magnetic moments due to orbiting electrons. To prove this statement, he designed an experiment with a rotating rod of steel inside a coil. Accelerated rod was expected to change the magnetic flux and therefore to produce current in the coil. The magnetization by rotation was confirmed. Today it is known as Barnett's effect. The reverse effect, i.e. rotation by magnetization was proven in the experiment performed by Einstein and de Haas and presented in [4]. They demonstrated that the angular momentum associated with an atomic system due to aligned elementary magnetic moments results in macroscopic rotation of the body. At present, quantum mechanics states that magnetization of matter is caused by oriented electron spins rather than their orbital momenta. In both mentioned experiments, ferromagnetic materials were used in order to obtain perceivable findings. These effects are so weak, that in no case can they explain magnetic fields of cosmic bodies, where, in addition, ferromagnetism cannot be expected. Nevertheless, they may provide seed magnetic fields necessary for the process of further generation. In the presented study, return to these original ideas is realized in order to proceed further in explanation of the phenomenon of cosmic magnetism. The considerations are related mainly to the case of Earth because its magnetic field is known best.

2.2 Earth's magnetic dipole moment

The geomagnetic field $\mathbf{B}$ on the Earth's surface can be expressed as the gradient of a scalar potential W,

$$\mathbf{B} = -\boldsymbol{\nabla} W, \tag{10}$$

whose further mathematical description is connected with the name of Johann Carl Friedrich Gauss. With his invention of spherical harmonic analysis, the geomagnetic potential has the following definition

$$W = R_E \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left(\frac{R_E}{r} \right)^{l+1} (g_l^m \cos m\varphi + h_l^m \sin m\varphi) \, P_l^m(\cos\vartheta), \tag{11}$$

where R_E is the radius of spherical Earth (6371 km), r, ϑ, φ are the spherical coordinates (radius, colatitude and longitude), P_l^m are the Schmidt-normalized associated Legendre functions of degree l and order m, and g_l^m and h_l^m are the Gauss coefficients. Through his analysis, mentioned for example in [5], Gauss showed that sources of the Earth's magnetic field lie within the body and that the dipole term

$$W_1 = \frac{{R_E}^3}{r^2}\left(g_1^0\cos\vartheta + g_1^1\sin\vartheta\cos\varphi + h_1^1\sin\vartheta\sin\varphi\right). \tag{12}$$

is dominant. Considering the scalar potential from the magnetic dipole moment $\boldsymbol{\mu}$ situated in the centre of Earth

$$W_d = \frac{\mu_0}{4\pi r^3}\,\boldsymbol{\mu}\cdot\mathbf{r} = \frac{\mu_0}{4\pi r^3}(\mu_i x_i + \mu_j x_j + \mu_k x_k) = \frac{1}{4\pi r^2}(\mu_i\sin\vartheta\cos\varphi + \mu_j\sin\vartheta\sin\varphi + \mu_k\cos\vartheta), \tag{13}$$

where x_k is the polar (rotational) axis, and x_i and x_j lie in the equatorial plane with x_i through 0^o longitude and x_j through 90^o East longitude, the Gauss coefficients of the degree $l = 1$ have this interpretation

$$g_1^0 = \frac{\mu_0}{4\pi {R_E}^3}\,\mu_k, \qquad g_1^1 = \frac{\mu_0}{4\pi {R_E}^3}\,\mu_i, \qquad h_1^1 = \frac{\mu_0}{4\pi {R_E}^3}\,\mu_j. \tag{14}$$

The Earth's magnetic moment can then be expressed as

$$\boldsymbol{\mu} = \frac{4\pi {R_E}^3}{\mu_0}(g_1^1\,\hat{\mathbf{i}} + h_1^1\,\hat{\mathbf{j}} + g_1^0\,\hat{\mathbf{k}}), \tag{15}$$

with $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ being Cartesian unit vectors in the directions of axes x_i, x_j and x_k, respectively. Its strength is about 8×10^{22} Am^2 and it is tilted by 11.5^o from the rotational axis pointing towards the geographic south. Since the geomagnetic field at the Earth's surface is predominantly dipolar, the magnetic moment (15) approximates it sufficiently well.

2.3 Magnetization by rotation

The Gauss' findings imply that there run some electromagnetic processes inside the neutral rotating Earth. They take place in the strongly electrically conducting core where conducting electrons are free to move in order to establish an internal balance. This balance must be in compliance with the fundamental principles of mechanics; conservation of momentum and conservation of angular momentum. To give rise to a dipole magnetic moment, the electrons should perform a collective circulatory motion about the same axis at a rate which highly exceeds the rotational rates of the whole body with which the electron system moves. The angular velocity of Earth generally consists of precession $\dot{\phi}$ and nutation $\dot{\theta}$ and the additional fast electron circulation represents spin $\dot{\psi}$. The spin does not affect the electrical neutrality of the core. Notional extraction of the electrons from the atom system leads to an idea of a body, which may be liken to a gyroscope. It is characterized by the electron (axially symmetric) distribution and by the spin $\dot{\psi}$ giving rise to an angular momentum along the gyroscope axis. The situation is illustrated in Figure 1. If $\boldsymbol{\omega}$ is the angular velocity of electrons, the overall angular momentum of the gyroscope is given by

$$\mathbf{L} = \int \rho\,\mathbf{r}\times\mathbf{v}\,\mathrm{d}V = \int \rho\,\mathbf{r}\times(\boldsymbol{\omega}\times\mathbf{r})\,\mathrm{d}V = m_e\int n_e\left[\boldsymbol{\omega}r^2 - \mathbf{r}(\mathbf{r}\cdot\boldsymbol{\omega})\right]\mathrm{d}V. \tag{16}$$

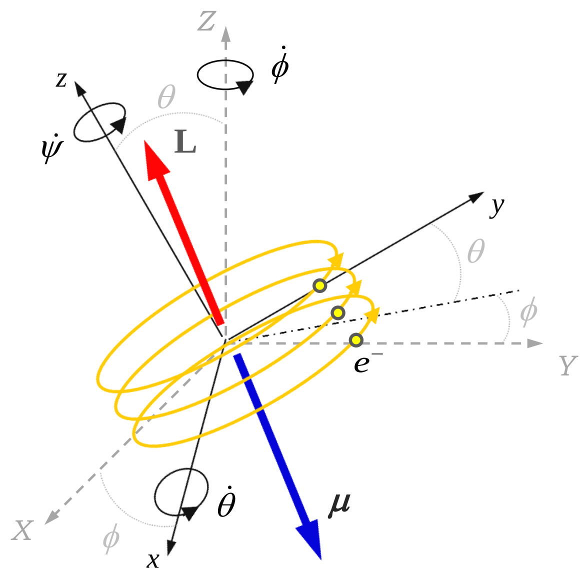

Figure 1: System of conducting electrons performing precession $\dot{\phi}$, nutation $\dot{\theta}$ and spin $\dot{\psi}$. For $\dot{\psi}$ highly exceeding $\dot{\phi}$ and $\dot{\theta}$, the angular momentum $\mathbf{L}$ is almost aligned with the spin axis.

Here $\mathbf{r}$ is the position vector from the centre of rotation, $\mathbf{v}$ is electron velocity due to the rotational motion, ρ is volume density of electrons, m_e is the electron mass and n_e is electron concentration. The dipole magnetic moment corresponding to this movement of charge is

$$\boldsymbol{\mu} = \frac{1}{2}\int \mathbf{r}\times\mathbf{j}\,\mathrm{d}V = \frac{1}{2}\int \mathbf{r}\times\rho_e\mathbf{v}\,\mathrm{d}V = \frac{1}{2}\int \rho_e\mathbf{r}\times(\boldsymbol{\omega}\times\mathbf{r})\,\mathrm{d}V = \frac{e^-}{2}\int n_e\left[\boldsymbol{\omega}r^2 - \mathbf{r}(\mathbf{r}\cdot\boldsymbol{\omega})\right]\mathrm{d}V, \tag{17}$$

with $\mathbf{j}$ being current density, ρ_e volume charge density and e^- the negative elementary charge. From comparison it is obtained

$$\boldsymbol{\mu} = \frac{e^-}{2m_e}\mathbf{L} = \gamma\,\mathbf{L}, \tag{18}$$

where $\gamma = e^-/2m_e$ is called the gyromagnetic ratio.

Behaviour of the gyroscope is going to be examined following Euler analysis of the rigid-body rotation presented in [6]. This is to be done in the Euler frame (x, y, z) derived from the inertial one (X, Y, Z) by rotations through the angles ϕ (the precession angle) and θ (the nutation angle);

$$\begin{aligned}
\hat{\mathbf{x}} &= \hat{\mathbf{X}}\cos\phi + \hat{\mathbf{Y}}\sin\phi, \\
\hat{\mathbf{y}} &= -\hat{\mathbf{X}}\cos\theta\sin\phi + \hat{\mathbf{Y}}\cos\theta\cos\phi + \hat{\mathbf{Z}}\sin\theta, \\
\hat{\mathbf{z}} &= \hat{\mathbf{X}}\sin\theta\sin\phi - \hat{\mathbf{Y}}\sin\theta\cos\phi + \hat{\mathbf{Z}}\cos\theta.
\end{aligned} \tag{19}$$

The angles ϕ and θ are instantaneous angles of the relative turn between the frames. The Euler frame rotates with respect to the inertial frame with the angular velocity $\boldsymbol{\Omega}$, which in the axes $x,\ y$, and z gets the form

$$\Omega_x = \dot{\theta}, \qquad \Omega_y = \dot{\phi}\sin\theta, \qquad \Omega_z = \dot{\phi}\cos\theta. \tag{20}$$

The axes $x,\ y,\ z$ are the principal axes of inertia of the rotating body. This fact allows to write

$$\mathbf{L} = I_{xx}\,\omega_x\,\hat{\mathbf{x}} + I_{yy}\,\omega_y\,\hat{\mathbf{y}} + I_{zz}\,\omega_z\,\hat{\mathbf{z}}, \tag{21}$$

in the rotating frame, where $\boldsymbol{\omega}$ is the angular velocity of the gyroscope and

$$\omega_x = \dot{\theta}, \qquad \omega_y = \dot{\phi}\sin\theta, \qquad \omega_z = \dot{\phi}\cos\theta + \dot{\psi}. \tag{22}$$

The quantities $I_{xx},\ I_{yy},\ I_{zz}$ are moments of inertia with respect to the $x,\ y$ and z axis. They are diagonal components of the inertia tensor $\bar{\bar{\mathbf{I}}}$ in principal axes and characterize electron distribution in the volume. In our case, the symmetry about the z-axis is supposed, therefore it can be written $I_{xx} = I_{yy} = I_0$ and $I_{zz} = I$. The gyroscope rotates with the frame and possesses the spin $\dot{\psi}$, which encompasses material properties of the atom system. For its angular momentum it is finally obtained

$$\mathbf{L} = I_0\,\dot{\theta}\,\hat{\mathbf{x}} + I_0\,\dot{\phi}\sin\theta\,\hat{\mathbf{y}} + I\,(\dot{\phi}\cos\theta + \dot{\psi})\,\hat{\mathbf{z}}. \tag{23}$$

Conservation of angular momentum gives

$$\left(\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right)_{inert} = \left(\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right)_{rot} + \boldsymbol{\Omega}\times\mathbf{L}. \tag{24}$$

Time change of $\mathbf{L}$ in the inertial frame is caused by sum of applied torques

$$\left(\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right)_{inert} = \boldsymbol{\tau}. \tag{25}$$

It means that if $\mathbf{L}$ is conserved in the rotating frame, there must be a torque $\boldsymbol{\tau}$, which balances the rotation term $\boldsymbol{\Omega}\times\mathbf{L}$. In short time scales (period associated with the rotation rate up to several years), the Earth's magnetic dipole moment seems to be in this situation. Even in longer times the dipole changes are perceivable only by precise measurements. Since this torque can be purely of electromagnetic nature, it can be deduced that the overall magnetic moment $\boldsymbol{\mu}$ is subject to a magnetic torque

$$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}. \tag{26}$$

Thus the balance

$$\boldsymbol{\mu}\times\mathbf{B} = \boldsymbol{\Omega}\times\mathbf{L} \tag{27}$$

is established, which in components looks like

$$\begin{aligned} \mu_y B_z - \mu_z B_y &= \Omega_y L_z - \Omega_z L_y, \\ \mu_z B_x - \mu_x B_z &= \Omega_z L_x - \Omega_x L_z, \\ \mu_x B_y - \mu_y B_x &= \Omega_x L_y - \Omega_y L_x. \end{aligned} \tag{28}$$

At the same time, it is presumed that the rotational movement about each of the axes causes the corresponding component of the magnetic moment as it is stated by the formula (18). Utilizing the expression (23) leads to equalities

$$\begin{aligned}
-\gamma\left(B_y\, I\,(\dot\phi\cos\theta+\dot\psi)-B_z\, I_0\,\dot\phi\sin\theta\right) &= \dot\phi\sin\theta\, I\,(\dot\phi\cos\theta+\dot\psi)-\dot\phi\cos\theta\, I_0\,\dot\phi\sin\theta,\\
-\gamma\left(B_z\, I_0\,\dot\theta-B_x\, I\,(\dot\phi\cos\theta+\dot\psi)\right) &= \dot\phi\cos\theta\, I_0\,\dot\theta-\dot\theta\, I\,(\dot\phi\cos\theta+\dot\psi),\\
-\gamma\left(B_x\, I_0\,\dot\phi\sin\theta-B_y\, I_0\,\dot\theta\right) &= 0.
\end{aligned} \tag{29}$$

Comparing the terms of both sides of the equations, it is found that

$$-\gamma B_y=\dot\phi\sin\theta,\qquad -\gamma B_z=\dot\phi\cos\theta,\qquad -\gamma B_x=\dot\theta, \tag{30}$$

i.e.

$$\mathbf{B}=-\frac{1}{\gamma}\mathbf{\Omega}. \tag{31}$$

Rotation of the gyroscope with preserving $\mathbf{L}$ (and $\boldsymbol{\mu}$) results in a magnetic field $\mathbf{B}$ depending linearly on $\mathbf{\Omega}$ by (31). $\mathbf{\Omega}$ is called the Larmor frequency. There is a related effect named by Barnett and presented in [3]; a spinning uncharged body forming an atom system that contains nonzero atomic magnetic moments (system paramagnetic or ferromagnetic) tends to spontaneously magnetize with magnetization $\mathbf{M}$ for which it holds

$$\mu_0\mathbf{M}=-\frac{\chi}{\gamma}\mathbf{\Omega}, \tag{32}$$

where $\chi=\mu_0 M/B$ is magnetic susceptibility of the material of the body. As regard to its magnetic effect, the rotation of the body is equivalent to an external field. Explanation of this fact can be also found in [7]. Although weak, this phenomenon gives a direct answer about the origin of cosmic magnetism.

3 Magnetic dipole moment and its changes

For simplicity, it is assumed that the electron distribution does not change in the rotating frame connected with the cosmic body, i.e. $(\mathrm{d}\bar{\bar{\mathbf{I}}}/\mathrm{d}t)_{rot}=0$. This allows to study behaviour of the gyroscope as a rigid body rotation. Provided that $\dot\phi$ and $\dot\psi$ are both nonzero, meaning that the body rotates and possesses a magnetic moment, validity of $(\mathrm{d}\mathbf{L}/\mathrm{d}t)_{rot}=0$ demands

$$\begin{aligned}
\left(\frac{\mathrm{d}L_x}{\mathrm{d}t}\right)_{rot} &= I_0\,\ddot\theta=0 &&\Rightarrow \ddot\theta=0,\\
\left(\frac{\mathrm{d}L_y}{\mathrm{d}t}\right)_{rot} &= I_0\,(\ddot\phi\sin\theta+\dot\phi\dot\theta\cos\theta)=0 &&\Rightarrow (\ddot\phi=0\ \wedge\ \dot\theta=0),\\
\left(\frac{\mathrm{d}L_z}{\mathrm{d}t}\right)_{rot} &= I\,(\ddot\phi\cos\theta-\dot\phi\dot\theta\sin\theta+\ddot\psi)=0 &&\Rightarrow (\ddot\phi=0\ \wedge\ \dot\theta=0\ \wedge\ \ddot\psi=0).
\end{aligned} \tag{33}$$

All these requirements are satisfied by the Earth's and also planetary magnetic dipole moments to large extent of precision, but not, for example, by the Sun's dipole moment.

$\ddot{\phi} = 0$ comes of changeless rotation of a cosmic body about the precessional axis. $\ddot{\psi} = 0$ is a feature of the ideal gyroscope. Its violation would mean a change in strength of the (measurable) magnetic moment as a consequence of a considerable change in internal physical-chemical conditions. $\dot{\theta} = 0$ means that the nutation angle θ between the Z-axis of the inertial frame ($\dot{\phi}$-axis) and z-axis of Euler frame (gyroscope or $\dot{\psi}$-axis) remains unchanged. This is the least reliable factor, which is never satisfied on Sun.

However, in light of long time scales (several years and longer), it is apparent that planetary magnetic moments are not conserved. The Earth's dipole moment is subject to several changes, the so called secular variations. Namely, there is the dipole axis motion and decrease in the dipole strength, both described in detail in [8]. The variation in the dipole strength indicates that the premise $(\mathrm{d}\bar{\bar{\mathbf{I}}}/\mathrm{d}t)_{rot} = 0$ is not perfectly fulfilled, which fact may be attributed to operation of some non-conservative torques. The first mentioned change consists mainly in the observed sustained westward motion of the north geomagnetic pole. There must be an additional torque responsible for it.

The gyroscope rotates not in vacuum, but with an ambient having some magnetic properties characterized by magnetic susceptibility χ. The total magnetic field arising from the rotational effects is then $\mathbf{B}(1 + \chi)$ and the additional magnetic torque

$$\boldsymbol{\tau}_\chi = \boldsymbol{\mu} \times \chi \mathbf{B} \tag{34}$$

is in action. The equation

$$\left(\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right)_{rot} = \boldsymbol{\mu} \times \chi \mathbf{B} \tag{35}$$

is going to be solved in spherical coordinates in order to relate the result to measurements carried on the Earth's surface and by satellites. The unit vectors of this new coordinate system are collinear with the unit vectors of the Euler system, and are obtained after the transformation

$$\begin{aligned}
\hat{\mathbf{r}} &= (-\hat{\mathbf{Y}})\sin\theta\cos\phi + \hat{\mathbf{X}}\sin\theta\sin\phi + \hat{\mathbf{Z}}\cos\theta \quad (= \hat{\mathbf{z}}), \\
\hat{\boldsymbol{\theta}} &= (-\hat{\mathbf{Y}})\cos\theta\cos\phi + \hat{\mathbf{X}}\cos\theta\sin\phi - \hat{\mathbf{Z}}\sin\theta \quad (= -\hat{\mathbf{y}}), \\
\hat{\boldsymbol{\phi}} &= -(-\hat{\mathbf{Y}})\sin\phi + \hat{\mathbf{X}}\cos\phi \quad (= \hat{\mathbf{x}}).
\end{aligned} \tag{36}$$

Here, the vector $\mathbf{L}$ can be considered as $\mathbf{L} = L\,\hat{\mathbf{L}}$ and its time change in the rotating frame is given by

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}L}{\mathrm{d}t}\,\hat{\mathbf{L}} + L\,\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t}, \tag{37}$$

where L stands for magnitude and the unit vector $\hat{\mathbf{L}}$ determines direction. The time change of L contains both, the time change of moments of inertia as well as the time changes of angular velocities, which would evidence accelerations of the frame. These will not be considered and only directional changes of $\mathbf{L}$ are of interest. The problem will then be reduced to determination of motion of the vector $\mathbf{L}$ in the rotating frame specified by the axes with the unit vectors $\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$. The vector components in this spherical coordinate system are, for $\mathbf{L}$

$$\begin{aligned}
L_r &= L_z &&= I\,(\dot{\phi}\cos\theta + \dot{\psi}), \\
L_\theta &= -L_y &&= -I_0\,\dot{\phi}\sin\theta, \\
L_\phi &= L_x &&= I_0\,\dot{\theta},
\end{aligned} \tag{38}$$

for $\boldsymbol{\mu}$

$$\begin{aligned} \mu_r = \quad & |\gamma| L_r \quad = |\gamma| I \, (\dot{\phi} \cos\theta + \dot{\psi}), \\ \mu_\theta = \quad & |\gamma| L_\theta \quad = -|\gamma| I_0 \, \dot{\phi} \sin\theta, \\ \mu_\phi = \quad & |\gamma| L_\phi \quad = |\gamma| I_0 \, \dot{\theta}, \end{aligned} \tag{39}$$

and for $\mathbf{B}$

$$B_r = -\frac{1}{\gamma} \dot{\phi} \cos\theta, \quad B_\theta = \frac{1}{\gamma} \dot{\phi} \sin\theta, \quad B_\phi = -\frac{1}{\gamma} \dot{\theta}. \tag{40}$$

Time derivative of a vector, say $\mathbf{a}$, in the spherical coordinate system is given by

$$\begin{aligned} \frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} \quad = \quad & \hat{\mathbf{r}} \left(\frac{\mathrm{d}a_r}{\mathrm{d}t} - a_\theta \frac{\mathrm{d}\theta}{\mathrm{d}t} - a_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta \right) + \hat{\boldsymbol{\theta}} \left(a_r \frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\mathrm{d}a_\theta}{\mathrm{d}t} - a_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \cos\theta \right) \\ & + \hat{\boldsymbol{\phi}} \left(a_r \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta + a_\theta \frac{\mathrm{d}\phi}{\mathrm{d}t} \cos\theta + \frac{\mathrm{d}a_\phi}{\mathrm{d}t} \right). \end{aligned} \tag{41}$$

Introducing the vector components (38), (39) and (40) into the equation (35), it gets the form of this system of equations

$$\begin{aligned} \frac{\mathrm{d}L_r}{\mathrm{d}t} - L_\theta \frac{\mathrm{d}\theta}{\mathrm{d}t} - L_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta &= 0, \\ L_r \frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\mathrm{d}L_\theta}{\mathrm{d}t} - L_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \cos\theta &= -\chi \left(-\dot{\phi} \cos\theta \, I_0 \, \dot{\theta} + \dot{\theta} \, I \, (\dot{\phi} \cos\theta + \dot{\psi}) \right), \\ L_r \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta + L_\theta \frac{\mathrm{d}\phi}{\mathrm{d}t} \cos\theta + \frac{\mathrm{d}L_\phi}{\mathrm{d}t} &= -\chi \left(\dot{\phi} \sin\theta \, I \, (\dot{\phi} \cos\theta + \dot{\psi}) - \dot{\phi} \cos\theta \, I_0 \, \dot{\phi} \sin\theta \right). \end{aligned} \tag{42}$$

If $\mathrm{d}L_r/\mathrm{d}t, \mathrm{d}L_\theta/\mathrm{d}t$ and $\mathrm{d}L_\phi/\mathrm{d}t$ are omitted, the equations can be arranged to

$$\begin{aligned} L_\theta \frac{\mathrm{d}\theta}{\mathrm{d}t} + L_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta &= 0, \\ L_r \frac{\mathrm{d}\theta}{\mathrm{d}t} + L_\phi \frac{\mathrm{d}\phi}{\mathrm{d}t} \cos\theta &= -\chi \dot{\theta} \left(L_\theta \frac{\cos\theta}{\sin\theta} + L_r \right), \\ L_r \frac{\mathrm{d}\phi}{\mathrm{d}t} \sin\theta + L_\theta \frac{\mathrm{d}\phi}{\mathrm{d}t} \frac{\cos\theta}{\sin\theta} &= -\chi \dot{\phi} \left(L_r + L_\theta \frac{\cos\theta}{\sin\theta} \right), \end{aligned} \tag{43}$$

from where it comes that

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\chi \dot{\theta} \tag{44}$$

and

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\chi \dot{\phi}. \tag{45}$$

Emphasize, that $\mathrm{d}\theta/\mathrm{d}t$ and $\mathrm{d}\phi/\mathrm{d}t$ bear on the position change of the vector $\mathbf{L}$ inside the rotating system, while $\dot{\theta}$ and $\dot{\phi}$ relate to the magnitude of its components. The results (44) and (45) are essential, because they describe the motion of the gyroscope (or the magnetic dipole) axis as dependences of the ambient parameter χ and the frame (cosmic body) rotation $\mathbf{\Omega}$. Implicitly they state that this motion is caused, contrary to the present belief, by external influence.

The study [8] presents the north geomagnetic pole latitudes and longitudes for years 1840-2000. Their behaviour is not as simple as prescribed by the formulas (44) and (45). This would suggest that though the gyroscope follows the Earth's rotation, probably it is also influenced by transfer of masses in the body and also in the atmosphere, and partially it may have its own regime. The most prominent and persistent feature is the westward motion of the north geomagnetic pole with the rate of about -0.05^o longitude per year (retrograde to the Earth's rotation). If this value is introduced into (45) and the Earth's rotation rate $\dot{\phi} = 360^o/(1\ yr/365)$ is taken, the value $\chi \approx 3.8 \times 10^{-7}$ is acquired. This corresponds with high precision to the magnetic susceptibility of air, when $\chi_{air} = 3.6 \times 10^{-7}$, as stated in [9]. It is not possible to deduce anything meaningful from (44), since Earth does not perform apparent nutation. From observations it is clear that the entire latitudinal motion of the north geomagnetic pole during the aforementioned time period is within the range of $\Delta\theta \sim 1°$, but it is hard to infer any definite trend. Irregularity and smallness of this motion is in compliance with the fact that $\dot{\theta} \approx 0$ for Earth.

4 Spin

To find out how the gyroscope is formed, i.e. how its spin is gained, a classical model of conducting electron gas is considered. A group of electrons being a part of a paramagnetic material (of the Earth's core), occupies some spherical volume of a cosmic body with a radius R. When the system does not rotate, an equilibrium is established

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = \mathbf{F}, \tag{46}$$

where the first term is time change of the net momentum $\mathbf{p} = m_e\mathbf{v}$, the second term represents collisions among electrons with the mean free time τ and $\mathbf{F}$ stand for some central force that keeps the electrons together. If the system is set into rotation with angular velocity $\mathbf{\Omega}$, according to what was said previously, the magnetic field $\mathbf{B}(1+\chi_s)$ is also present. $\chi_s \neq 0$ is now magnetic susceptibility of the concerned material. In the simplest case it is a mere constant and its sign depends on whether the matter is paramagnetic or diamagnetic. The new situation is described by the balance

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = \mathbf{F} + 2m_e(\mathbf{v} \times \mathbf{\Omega}) + e^-(\mathbf{v} \times \mathbf{B}) + e^-(\mathbf{v} \times \chi_s\mathbf{B}) - m_e\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}). \tag{47}$$

Making reference to Larmor's theorem, see for example [10], which says that the motion with a magnetic field is always one of the no-field solutions with an added rotation, and taking (32) into account, it is obtained

$$-2m_e(\mathbf{v} \times \chi_s\mathbf{\Omega}) - m_e\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) = 0. \tag{48}$$

Usually, both terms in (48) are considered very small and therefore negligible when dealing with the primary balance of electron systems in different contexts. In the case of

Earth, where $\mathbf{\Omega} = \dot{\phi}\hat{\mathbf{Z}}$, it is convenient to study this secondary balance in the cylindrical geometry (s, ϕ, Z). In ϕ-direction it is then obtained $v_s \Omega = 0$, so

$$v_s = 0. \tag{49}$$

In s-direction it holds $-2m_e\, v_\phi\, \chi_s\, \Omega + m_e\, \Omega^2 s = 0$, from where

$$v_\phi = s\, \frac{\Omega}{2\chi_s}. \tag{50}$$

The sustained flow of negative charge resulting from (48) happens in the azimuthal direction and as such, it causes a retrograde azimuthal electric current. Furthermore, if $v_\phi = s\, \dot{\psi}$,

$$\dot{\psi} = \frac{\Omega}{2\chi_s} \tag{51}$$

represents the searched spin of the gyroscope. It depends on magnetic properties of the material represented by χ_s and on largeness of the rotation Ω. It is a direct consequence of the Barnett's effect. For $\chi_s > 0$, like in the Earth's core, it has the same orientation as $\mathbf{\Omega}$. Unless the material is ferromagnetic, in which case different physics applies, $\dot{\psi}$ highly exceeds Ω fulfilling the necessary condition for the gyroscope. It eventuates in the angular momentum along the spin axis associated with the magnetic moment pointing in the opposite direction. This particularity is in compliance with the main attribute of the Earth magnetic field. It can be confirmed using the Biot-Savart-Laplace law. There is the magnetic field in the center of the sphere from currents $I\mathrm{d}\mathbf{l} = \mathbf{j}\,\mathrm{d}V$, $\mathbf{j} = -j\hat{\boldsymbol{\phi}}$, running inside the volume

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{\mathrm{d}\mathbf{l} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} j \frac{(-R\sin\theta\, \mathrm{d}\phi\hat{\boldsymbol{\phi}}) \times (-R\,\hat{\mathbf{r}})}{R^3} R\,\mathrm{d}\theta\,\mathrm{d}r = \frac{\mu_0}{4\pi} j\, \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}r\,\hat{\boldsymbol{\theta}}. \tag{52}$$

Considering that $\hat{\boldsymbol{\theta}} = \hat{\mathbf{s}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ and $j = const.$, the total magnetic field is given as

$$\mathbf{B} = -\frac{\mu_0}{4\pi} j \int_0^R \mathrm{d}r \int_0^\pi \sin^2\theta\,\mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi\,\hat{\mathbf{z}} = -\frac{\mu_0 \pi R\, j}{4}\hat{\mathbf{z}}. \tag{53}$$

It is unknown how much charge Q participates on this proposed mechanism of magnetic field creation, so one cannot estimate B from (53). One thing that can be done is to use (53) to compare individual known magnetic fields exhibiting similar features and emergence conditions. In Solar system, the only planetary magnetic field showing the substantial similarity with the one of Earth's, is the Mercury's field. These two planets have the same sense of rotation and the same polarity of their prevailingly dipolar magnetic fields. Both possess significantly large iron cores where the magnetic fields are generated. For the current density j in (53) it can be written

$$j = \frac{Q}{S_\phi}\frac{1}{t} = \frac{\rho_e V}{\int_0^R \int_0^\pi r\mathrm{d}\theta\mathrm{d}r}\frac{\dot{\psi}}{2\pi} = \frac{4R\rho_e\dot{\psi}}{3\pi} \tag{54}$$

leading to the magnetic field magnitude

$$B = \frac{\mu_0 \rho_e R^2 \dot{\psi}}{3} = \frac{\mu_0 \rho_e R^2 \Omega}{6\chi_s}. \tag{55}$$

Assuming similar material properties and taking the values of the core radii for Earth and Mercury, $R_{♁} = 3480$ km, $R_{☿} = 1800$ km, respectively, and $T_{♁} = (1/58.81)T_{☿}$ for their rotation periods, gives

$$\frac{B_{☿}}{B_{♁}} = \frac{R_{☿}^2 T_{♁}}{R_{♁}^2 T_{☿}} = 4.55 \times 10^{-3}. \tag{56}$$

This ratio is in a good agreement with measurements made by NASA space probe Messenger stating $B_{☿} \approx 0.006 B_{♁}$.

In real situations in Solar system, the $\dot{\psi}$-axis is not aligned with the planetary rotation axis, but generally it is declined at some angle. The only exception is Saturn's magnetic field showing no tilt and almost perfect axisymmetry. This fact poses a difficulty for the dynamo theory in attempt to explain its creation. The asymmetry infers that the balance (48) is perturbed. A possible explanation is anisotropy of magnetic properties resulting probably from the physical-chemical properties of the atom system. It holds especially if the material is at least partially crystallized. This concerns both mentioned, Earth's and Mercury's iron cores. Moreover, the tensor character of χ_s may be responsible for appearance of a non-dipole part of planetary magnetic fields, which is markedly present in the fields of Uranus and Neptune. Both of these planets embody inverse polarity of their magnetic fields with respect to orientation of their rotational axes as compared to the Earth's one. This fact may indicate diamagnetism of the region (the water-ammonia ocean) where the fields are generated or, more probably (because atomic diamagnetism is a very weak effect), positive free charge carriers. There are scientific hypotheses proposed for example in [11], that under extreme conditions deep within Uranus and Neptune, the water is in a superionic state. In this state the oxygen crystallizes but the hydrogen ions move freely within the oxygen lattice. Jupiter and Saturn show similar polarity, however their diamagnetism is of different nature. Metallic hydrogen composing part of their interiors is considered to be a high temperature superconductor, as it was suggested in [12]. Its diamagnetism is thus not of atomic origin but a consequence of electron currents compensating the 'external' magnetic field. Again, in this special case the equation (48) is of little use because the classical electron gas model cannot be applied. Instead, electrons form coupled (Cooper) pairs leading to a cooperative behaviour, see [13], and their current flows with a density exponentially decreasing inwards, as it was proven in [14]. This problem requires an individual approach to be resolved.

Note, that thanks to applying the classical electron gas (Drude) model, there were not any additional requirements for magnetic field to be generated, such as heating and character of heat sources, liquid state of the concerned matter and unavoidable asymmetry with respect to the rotational axis. All these are conditions for a workable astrophysical dynamo pursuant to the dynamo theory. For the gyroscope as a magnetic field producer, merely free charge carriers must be present and sufficiently fast rotation of the cosmic body. In this connection a question regarding the absence of Mars' magnetic field arises. Mars also possesses a core composed mostly of iron with the radius of about 1700 km and rotates about its axis with almost the same rate like Earth does. According to the formula (55), it should thus have a magnetic field with the magnitude

$B_{♂} \approx 0.23 B_{♁}$. The reason why it is not observed may consist in a considerably low temperature (the temperature of the core is only about 1500 K) due to which the material of the rest of the planet recovers strong magnetic properties. Thanks to abundance of iron in the relatively thick mantle and crust, magnetic shielding becomes possible at temperatures above the Curie (Néel) point. This eventuality could also explain strong magnetization of the crust material.

5 Polarity reversal

It was shown that the rotating gyroscope formed from charged particles preserves its angular momentum and the corresponding magnetic dipole moment in the rotating frame in agreement with the perfect balance expressed by (27). To break this balance and obtain - in the extreme case - a flip over of the magnetic moment, an additional torque is needed. It can be applied with a weak magnetic field from the side, following the principle described by Feynman in [10]. In astrophysical gyroscopes, this additional torque can be achieved by the intrinsic magnetic fields of rotating cosmic bodies thanks to Barnett's effect. It has been already given by the formula (34). To gain the required impact, a lateral component of rotation of the ambient (with nonzero χ) corotating with the gyroscope about the vertical axis would be necessary. If a cosmic body rotates differentially about a vertical axis, the desired lateral component of $\mathbf{\Omega}$, namely $\dot{\theta}$, is also present. It leads to magnetization $M_x = -\chi\dot{\theta}/(\mu_0\gamma)$ according to the formula (32), present in the frame rotating about the vertical axis. It gives rise to non-radial forces causing the balance (47) and subsequently also (48) to be disturbed. Recalling the equation (44) for the meridional motion of the dipole axis, and taking $\Delta\theta = \pi$ and $\Delta t = T_{osc}/2$, the ratio

$$\frac{\Delta\theta}{\Delta t} = \omega_R \tag{57}$$

constitutes the equivalent of Rabi frequency of oscillation of a quantum mechanical two-level system.

The dipole moment polarity reversal is best observed at Sun's magnetic field. It is part of a very regular process called solar cycle with the period 22 years. Such frequent destruction and follow-up recovery is probably responsible for the fact that Sun's dipole magnetic field is relatively weak, only about twice the Earth's one. The solar cycle relates to the surface activity in the convective zone resulting in complex series of local magnetic fields observed as sunspots. They vary over time and highly (thousands of times) exceed the main dipole field, which is believed to be created in tachocline just below the convective zone. Since plasma cannot be treated as a magnetic medium, see for example [16], the previous discussion about the lateral magnetization does not concern this case. Instead, magnetohydrodynamic effects take place. It is believed that differential rotation is the underlying cause of the magnetic ropes on the Sun. They provide an active lateral magnetic field necessary for the dipole to be flipped. Additionally to the basic differential rotation, a torsional wave pattern with alternating latitude zones of slow and fast rotation has been observed. It takes 22 years for the zones to drift from poles to the equator what comprises the period of the full solar cycle and prove the connection with magnetic inversions. At solar activity minimum, the number of sunspots is reduced and the dipole field is parallel or antiparallel with the rotation axis. The details of these coexistent processes are given in [15].

6 Conclusions

A new mechanism of creation of large-scale cosmic magnetic fields has been proposed. It is based on gyroscopic motion of free charge carriers which form a part of atomic systems composing rotating bodies. To obtain this collective behaviour, besides a sufficiently rapid rotation only weak magnetic properties of the concerned material are required. The latter demand is always satisfied in the interiors of cosmic bodies. The gyroscopic motion of charge results in a magnetic moment and as such can well account for existence of astrophysical magnetic fields. If a magnetic torque is applied by some external influence, the magnetic moment undergoes a change, which in the extreme case may lead to a magnetic reversal. The Barnett's effect of magnetization of the non-conducting parts of rotating bodies provides a torque, whose impact is normally evident only in large time scales. Relating it to the Earth's field, this conception naturally explains the observed slow westward motion of the north geomagnetic pole. Measurements of this kind of secular changes could help to assess material composition of cosmic bodies and their atmospheres within the meaning of the parameter χ. It was also stated that varied asymmetry of magnetic dipole moments with respect to axes of rotation of cosmic bodies may be ascribed to magnetic anisotropy of their interiors. Detailed determination of configurations of magnetic fields could then provide valuable information about internal planetary structure. To find answers for each particular case, deep study comprising accessible knowledge of condensed matter physics and astrophysics is required.

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