The harmonic inversion method: calculation of the multi-domain density

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A b s t r a c t : An improved variant of the harmonic inversion method for solving the inverse gravimetric problem is presented. The improvement with respect to the original variant consists of the possibility to calculate the position and shape of many anomalous bodies at once. The solution is sought in the form of a multi-domain density distribution. The calculation begins by placing the germs of future anomalous bodies into the model containing only a set of infinite horizontal layers with constant density; the germs are placed at the points of local extrema of the χ-density that is calculated from the measured surface gravitational field. In an iterative procedure the shape of anomalous bodies is changed until they acquire their final form; in this process the residual χ-density plays a key role. The results of the numerical calculation are discussed.

Key words: gravity inversion, anomalous body, χ-density, integral transformation, iteration

1. Introduction

The harmonic inversion method was used for the first time by the calculation of the shape of the Kolárovo anomalous body (see *Pohánka (2001)*). However, this original variant of the method enabled to calculate only the shape of a single body at once; if we would require to determine the shape of more anomalous bodies, the calculation had to be sequential (thus the already determined body shapes could not be changed by the calculation of the shape of the next body). Moreover, the shape of an anomalous body could not be quite arbitrary, as for any vertical line, the body could have at most one common line segment with this line. Therefore it was necessary to find a procedure which would enable to find the shapes of an arbitrary

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number of anomalous bodies at once; this advanced variant of the harmonic inversion method is presented here.

2. Properties of the characteristic density

The main tool for solving the inverse gravimetric problem according to the *harmonic inversion method* is the characteristic density (or shortly χ-density). Recall that the present variants of the harmonic inversion method are suitable only for the case that the earth surface is a plane (we define it as the plane $z = 0$); any density distribution can be nonzero only below this plane (thus for $z \leq 0$). We first introduce the integral transformation

$$F(x, y, z) = T(x, y, z)\, f(*, *), \tag{1}$$

defined for any sufficiently smooth function $f(x, y)$ for $z \geq 0$ as

$$T(x, y, z)\, f(*, *) = 0 \tag{2}$$

and for $z < 0$ as

$$T(x, y, z)\, f(*, *) = \frac{20}{\pi\, \kappa} \int_0^\infty \mathrm{d}u\, \frac{u^4\, z^4}{(u^2 + z^2)^{7/2}}\, \partial_u \frac{1}{u}\, \partial_u\, \bar{f}(x, y, u), \tag{3}$$

where

$$\bar{f}(x, y, u) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi\, f(x + u\, \cos\varphi, y + u\, \sin\varphi) \tag{4}$$

and κ is the gravitational constant. If we denote the surface gravitational effect (which is equal to minus vertical component of the gravitational acceleration at the surface) as $a(x, y)$, the formula for the χ-density corresponding to this gravitational effect reads

$$\chi(x, y, z) = T(x, y, z)\, a(*, *). \tag{5}$$

Consider any density distribution $\rho(x, y, z)$ (nonzero only for $z \leq 0$); this density generates the surface gravitational effect $a(x, y)$ described by the well known integral transformation

$$a(x, y) = S(x, y)\, \rho(*, *, *), \tag{6}$$

where

$$S(x,y)\, f(*,*,*) = \kappa \int_{z' \leq 0} \mathrm{d}V'\; G(x-x', y-y', -z')\; f(x', y', z') \tag{7}$$

and

$$G(x,y,z) = \frac{z}{(x^2+y^2+z^2)^{3/2}}\,. \tag{8}$$

Using the formulae (5) and (6), for any given density distribution $\rho(x,y,z)$ we obtain the corresponding χ-density $\chi(x,y,z)$; on the contrary, for any χ-density $\chi(x,y,z)$ there exist infinitely many density distributions $\rho(x,y,z)$ generating this χ-density, as there exist infinitely many density distributions $\rho(x,y,z)$ generating the same surface gravitational effect $a(x,y)$.

In the case that the surface gravitational effect $a(x,y)$ is a constant, from (3) and (4) we easily obtain that the corresponding χ-density is identically zero. This means that for any density distribution generating a constant surface gravitational effect, the corresponding χ-density is identically zero. Particularly, this is the case for the density distribution describing a set of infinite horizontal layers such that the density within each layer is a constant.

Recall that the χ-density $\chi(x,y,z)$ as a function of the given surface gravitational effect $a(x,y)$ is defined as the density distribution generating this gravitational effect (up to a constant) and satisfying the following conditions:

1. It is a maximally smooth function (it holds $\Delta^k \chi(x,y,z) = 0$ for the smallest possible k).

2. It is a linear integral transformation of the surface gravitational effect.

3. For the surface gravitational effect generated by a point source lying below the surface, it has its main extremum at the point source.

These conditions define the χ-density uniquely (in the form (5)) and the funtion $\chi(x,y,z)$ has for $z<0$ always the form $z^3 h(x,y,z)$, where $h(x,y,z)$ is a harmonic function (thus the smallest value of k from condition 1 is equal to 4).

If the surface gravitational effect $a(x,y)$ tends to zero at infinity (this is the case, for example, if the density $\rho(x,y,z)$ generating this effect is nonzero only in a bounded domain), the corresponding χ-density $\chi(x,y,z)$ is a solution of the inverse problem for this surface gravitational effect, and

$$a(x,y) = S(x,y)\,\chi(*,*,*). \tag{9}$$

In the case of the gravity field generated by a point source with mass m_0 located at the point $(x_0,y_0,-d_0)$ (where $d_0 > 0$), the function $\chi(x,y,z)$ has the main local extremum at the same point and the value of this extremum is $5m_0/2\pi {d_0}^3$. The same surface gravitational effect as this point source has the spherical inhomogeneity with centre at the point source, radius r_0 (such that $r_0 < d_0$) and density ρ_0 such that $m_0 = 4\pi\rho_0 {r_0}^3/3$. This is the one important case, when the whole set of anomalous bodies has the same corresponding χ-density.

More generally, consider the following class of anomalous bodies: each body is represented by a simply connected domain located in the halfspace $z \leq 0$; the boundary of the domain is sufficiently smooth; the density within this domain is a constant and outside is equal to zero. For any such body there exists a whole set of bodies from this class, generating the same surface gravitational effect as the former body (thus each body has the same corresponding χ-density). The densities of bodies from this set lie within some interval and for each value of the density from this interval there exists a single body; with decreasing absolute value of the density the size of the body increases until the boundary of the body touches the plane $z = 0$.

3. Determination of the multi-domain density

The χ-density is a solution of the inverse gravimetric problem, but it is surely not a realistic solution: it is a very smooth function of position. As it was noted already in *Pohánka (2001)*, the realistic distribution of density below the earth surface should be rather a partially constant function; in other words, the space below the surface is divided into several domains such that the density within each domain is a constant. We shall call such density distribution shortly the multi-domain density. The problem is to find a solution of the inverse problem that is a multi-domain density.

This problem is solved in the iterative way: some initial multi-domain density is chosen and this density is iteratively changed until it represents (to some approximation) the solution of the inverse problem. From the previous discussion it is clear that the change of the multi-domain density means the change of the shape of the particular domains, while the densities

within the domains have to be held unchanged. The change of the densities of the domains would mean that we look for another solution of the inverse problem. Of course, it is quite possible that for some choice of the values of density in the particular domains, there needs not exist a solution of the inverse problem for any choice of the shape of the domains.

According to the previous section, infinite horizontal layers with densities constant within each layer are not detectable by the χ-density. This means that we have to choose some multi-domain density distribution whose domains will be such horizontal layers; we shall call this density distribution the zero model (as the cooresponding χ-density of this density distribution is zero). The zero model is defined by the depths of the boundaries of the layers (the uppermost layer has its upper boundary at the zero depth) and by the densities within particular layers. Any multi-domain density will be then constructed from this zero model by introducing some number of additional domains to the zero model (together with some values of density for each additional domain); such multi-domain density will be called a model.

If we denote the density distribution of the zero model as $\rho_0(x, y, z)$ and the density distribution of the model as $\rho_m(x, y, z)$, we can calculate the surface gravitational effect of the model $a_m(x, y)$ as

$$a_m(x, y) = S(x, y)\,(\rho_m(*, *, *) - \rho_0(*, *, *)), \tag{10}$$

and the corresponding χ-density of the model $\chi_m(x, y, z)$ as

$$\chi_m(x, y, z) = T(x, y, z)\,a_m(*, *). \tag{11}$$

This means that in the calculation we use the difference density with respect to the zero model (this model contributes to the surface gravitational effect by a constant that can be neglected; moreover, if the density in the lowermost layer of the zero model is nonzero, this constant is infinite). If this difference density is nonzero only in a bounded domain, we have no problems by the calculation of the integral on the rhs of formula (10) and also the numerical calculation is simpler.

Note that the use of difference density (with respect to the zero model) has the following consequence: for an anomalous body represented in the model by some (bounded) domain and a constant density within this domain, if this domain intersects more than one layer of the zero model, the surface gravitational effect of this body is equal to the effect of a layered

body, where the density in each layer is equal to the difference of the density of the body and the density in the particular layer of the zero model (outside the body the difference density is zero). This shows that the choice of the zero model has a great importance for the solution of the inverse problem.

In the case of a single domain with constant value of difference density, if the horizontal dimensions of this domain are not much greater than its vertical dimension, the χ-density corresponding to the surface gravitational effect generated by this domain is substantially different from zero only within this domain and in its nearest neighbourhood. The main extremum of this χ-density lies in the domain (usually near its centre of mass) and the sign of this main extremum (and usually also the sign of the χ-density in the whole domain) is the same as the sign of the difference density in the domain. As a consequence, in the general case, for any significant local extremum of the χ-density there has to exist some domain of the multi-domain density containing the point of this extremum and the sign of the difference density in this domain has to be the same as the sign of this extremum. Of course, it may happen that there have to be in fact several domains with different difference densities, but it is impossible that there would be no domain (with nonzero difference density) in the neighbourhood of this extremum. This fact is used by the construction of the starting model: for every significant local extremum of the χ-density this model has to contain some domain containing this extremum and having the correct sign of the difference density.

For the iterative calculation of the model that is a solution of the inverse problem we shall use the residual surface gravitational effect of the model $a_{rm}(x, y)$ defined by

$$a_{rm}(x, y) = a(x, y) - a_m(x, y), \tag{12}$$

where $a(x, y)$ is the original (measured) surface gravitational effect and $a_m(x, y)$ is given by (10), and the residual χ-density of the model $\chi_{rm}(x, y, z)$ defined by

$$\chi_{rm}(x, y, z) = \chi(x, y, z) - \chi_m(x, y, z), \tag{13}$$

where $\chi(x, y, z)$ can be calculated according to formula (5) and $\chi_m(x, y, z)$ is given by (11); in fact we shall calculate the quantity $\chi_{rm}(x, y, z)$ using the formula

$$\chi_{rm}(x,y,z) = T(x,y,z)\, a_{rm}(*,*). \tag{14}$$

If the multi-domain density $\rho_m(x,y,z)$ would be a solution of the inverse problem, the residual surface gravitational effect $a_{rm}(x,y)$ would be a constant and the residual χ-density $\chi_{rm}(x,y,z)$ would be identically zero. Consequently, if the residual surface gravitational effect $a_{rm}(x,y)$ is not identically constant or if the residual χ-density $\chi_{rm}(x,y,z)$ is not identically zero, then $\rho_m(x,y,z)$ is not a solution of the inverse problem. Both functions $a_{rm}(x,y)$ and $\chi_{rm}(x,y,z)$ represent in some sense a measure of the distance of the model density $\rho_m(x,y,z)$ from a solution of the inverse problem. However, there is a great difference between these two measures: the difference between the model density $\rho_m(x,y,z)$ and a solution of the inverse problem at the point (x,y,z) is closely related to the behaviour of the residual χ-density in the neighbourhood of the point (x,y,z), but nothing similar can be said about the residual surface gravitational effect. This means that the procedure of changing the model density has to be based on the requirement to lessen the absolute values of the residual χ-density.

Although it is hardly possible to prove it exactly, several numerical calculations have shown the correctness of the following procedure: for every point (x,y,z) lying on the boundary of any two domains of the model, if the residual χ-density $\chi_{rm}(x,y,z)$ is positive (negative), the boundary in the small neighbourhood of this point should be shifted in the direction towards the domain whose density is smaller (greater) than the density of the other domain; in other words, the domain with greater (smaller) density should expand, while the other domain should shrink.

For the actual calculation we choose the calculation domain, usually of the form of a rectangular prism whose upper boundary lies at the surface. We divide this domain into a net of elementary rectangular prisms, usually of the form of a cube (this will be for simplicity assumed here); these elementary prisms will be called cells. Let the length of the edge of each cell be s_d; we choose the rectangular coordinate system x,y,z with axes parallel to the edges of the calculation domain and let the sides of the calculation domain lie in the planes $x = x_a$, $x = x_b$, $y = y_a$, $y = y_b$, $z = 0$, $z = -d_b$, where $x_b = x_a + n_x s_d$, $y_b = y_a + n_y s_d$, $d_b = n_d s_d$, and n_x, n_y, n_d are positive integers. The cells will be denoted by the triples (i,j,k), where $1 \le i \le n_x$, $1 \le j \le n_y$, $1 \le k \le n_d$; the centre of the cell (i,j,k) will be at the point

$(x_{c,i}, y_{c,j}, -d_{c,k})$, where

$$x_{c,i} = x_a + (i - 1/2)s_d, \;\; y_{c,j} = y_a + (j - 1/2)s_d, \;\; d_{c,k} = (k - 1/2)s_d.$$

For every cell of the calculation domain we define its neighbouring cells: for the cell (i, j, k) it will be the cells $(i-1, j, k)$, $(i+1, j, k)$, $(i, j-1, k)$, $(i, j+1, k)$, $(i, j, k-1)$, $(i, j, k+1)$ (if such cells exist). In order to assure that every cell of the calculation domain will have exactly 6 neighbouring cells, we add to the calculation domain at each side one layer of cells; this extended calculation domain will thus contain cells (i, j, k) for $0 \le i \le n_x+1$, $0 \le j \le n_y + 1$, $0 \le k \le n_d + 1$.

Each model will be defined by assigning to every cell (i, j, k) of the calculation domain some (positive) value of density; thus the density distribution $\rho_m(x, y, z)$ will be a function constant within each cell of the calculation domain. The model will be then defined for each cell of the extended calculation domain (this will be described below).

The input of the inverse problem is represented by the measured surface gravitational effect $a(x, y)$; this function has to be given (at least in a sufficiently dense net of points) in the input domain, usually a rectangle on the earth surface containing the upper boundary of the calculation domain. Here we shall assume that the function $a(x, y)$ is known at every point $(x_{c,i}, y_{c,j})$, where $1 - n_e \le i \le n_x + n_e$, $1 - n_e \le j \le n_y + n_e$, and n_e is sufficiently greater than n_d (in practice this means that $n_e \ge 5n_d$). The set of these points of the input domain will be called shortly the input net.

Now we can describe in detail the algorithm of calculation of a solution of the inverse problem in the form of a multi-domain density. We first choose the zero model (see above): for every k (such that $1 \le k \le n_d$) we choose the value of density $\rho_{0,k}$ and then assign to every cell (i, j, k), $0 \le i \le n_x + 1$, $0 \le j \le n_y + 1$, the density $\rho_{0,k}$. Then for every i, j, $0 \le i \le n_x + 1$, $0 \le j \le n_y + 1$, we assign to the cell $(i, j, 0)$ the density of the cell $(i, j, 1)$ and to the cell $(i, j, n_d + 1)$ the density of the cell (i, j, n_d). In this way the density is assigned to every cell of the extended calculation domain and the function $\rho_0(x, y, z)$ is defined.

The next step is the calculation of the χ-density $\chi(x, y, z)$ from the measured surface gravitational effect $a(x, y)$ according to formula (5); the function $\chi(x, y, z)$ should be calculated at the centre of each cell of the calculation domain from the values of the function $a(x, y)$ at the points of the

input net. In this way every cell of the calculation domain has assigned its value of χ-density; for the cells of the extended calculation domain that do not belong to the calculation domain, we assign the zero value of χ-density.

Now we construct the starting model for the subsequent iterative calculation; the density distribution in the I-th iteration will be denoted as $\rho_{m,I}(x, y, z)$ and the starting model represents the zeroth iteration. We first find each cell of the calculation domain such that the χ-density has a local extremum at this cell (such cell will be called the extremal cell): this is done by comparing the χ-density of each cell with the χ-densities of its neighbouring cells. For every extremal cell we can change the density of this cell to a new (positive) value; the difference of the new and the original value has to have the same sign as the χ-density of the extremal cell, but otherwise the new value of the density can be arbitrary. Several numerical calculations have shown that the suitable value for the difference of the new and the original density is of the same order as (or even equal to) the χ-density of the extremal cell. Of course, the new value of density can be restricted to some interval from a physical point of view and the most favourable case is to have some a priori information about the the real distribution of density in the particular domain.

Every extremal cell that has acquired in this way the new value of density will be called a germ (of the future anomalous body) as it represents a new domain of the multi-domain density that will usually grow in size in the subsequent iterative calculation. After we have created all germs, the density distribution of the starting model $\rho_{m,0}(x, y, z)$ is defined and we can calculate according to the formula (10) the surface gravitational effect of the starting model (at each point of the input net)

$$a_{m,0}(x, y) = S(x, y)\,(\rho_{m,0}(*, *, *) - \rho_0(*, *, *)) \tag{15}$$

and then also the corresponding residual surface gravitational effect

$$a_{rm,0}(x, y) = a(x, y) - a_{m,0}(x, y). \tag{16}$$

Note that in the formula (15) we have to consider by the integration only the cells that are germs (for other cells the difference of densities on the rhs is zero).

Finally we can start the iterative procedure for finding the multi-domain density that is a solution of the inverse problem. We shall describe here

the construction of the $(I+1)$-th iterative model from the I-th model: we already know the density distribution $\rho_{m,I}(x,y,z)$ and the corresponding residual surface gravitational effect $a_{rm,I}(x,y)$. For every cell of the calculation domain (in the I-th model) we determine, whether the densities of all 6 neighbouring cells are the same as the density of this cell; if this is the case, the cell will be called the interior cell, otherwise it will be called the boundary cell. The density of a cell can be changed only if it is a boundary cell because the new density can be only one from among the densities of its neighbouring cells. For every boundary cell of the I-th model we first calculate the residual χ-density at the centre of the cell from the residual surface gravitational effect $a_{rm,I}(x,y)$ using the formula (14)

$$\chi_{rm,I}(x,y,z) = T(x,y,z)\, a_{rm,I}(*,*). \tag{17}$$

The density of the boundary cell can be changed only if this value is nonzero; if this residual χ-density is positive (negative), we test whether there is among the densities of the neighbouring cells at least one density that is greater (smaller) than the density of the considered boundary cell. If we find at least one such density, we choose the density that is the closest to the density of the considered boundary cell, and this density can be assigned as the new density of this boundary cell; otherwise the density of this boundary cell remains unchanged. The choice of the nearest possible value of density has to assure that the changes of the model are as small as possible.

Whether the new value of density will be actually assigned to the considered boundary cell depends on the absolute value of the residual χ-density at the centre of this cell. If we change the density of the single cell (i,j,k) by δ, the mass of the cell will change by $\delta {s_d}^3$ and (if we approximate the surface gravitational effect of the cell by the effect of a point source located at the centre of the cell, thus at the depth $(k-1/2)s_d$) the corresponding χ-density of the model will change at the centre of this cell by a value approximately equal to $5\delta/2\pi(k-1/2)^3$ and the corresponding residual χ-density of the model will change by minus this value. We require that the change of the density of a cell is performed only if the new residual χ-density is absolutely smaller than the original residual χ-density. As the sign of δ is the same as the sign of the original residual χ-density, this condition will be satisfied only if the original residual χ-density is absolutely greater than $5|\delta|/4\pi(k-1/2)^3$.

As in the most cases the density is changed not only for a single separated cell but for many cells at the boundary of a domain, the limiting value for the residual χ-density has to be in fact greater. It is hardly possible to determine exactly what effect the change of density of many cells would have, but from the properties of the χ-density (as a function of the surface gravitational effect) we can conclude that the number of cells which should change their density is roughly proportional to the square of the depth (as the boundary of a domain is a 2-dimensional object). Therefore it is reasonable to require that the density of a boundary cell is changed only if the residual χ-density at the centre of this cell is greater than the limiting value equal to $c|\delta|/(k-1/2)$, where c is a suitable constant with value of order 1.

By the above described way of constructing the next iteration model the boundary of two domains with different densities is shifted in the direction of one of these domains by at most one cell; this happens regardless of the absolute value of the residual χ-density of the boundary cells (if all other conditions are satisfied). This is because it is almost impossible to find an exactly defined way to shift the boundary by more cells at once. In order to improve the reliability of the iteration procedure, it is suitable to change the above construction in such a way that the priority for changing their density would be for those boundary cells whose absolute value of their residual χ-density is the greatest. This can be done by imposing another limiting value for the change of the density of boundary cells; this limiting value should slowly decrease with the index I of iteration and its value in the zeroth iteration should be sufficiently great in order to allow the change of density of only a small number of cells. The exact form of this second limiting value can be determined experimentally by performing the numerical calculation.

After assigning the new values of density for all boundary cells according to the above mentioned criteria, the density distribution $\rho_{m,I+1}(x,y,z)$ is defined, and from equations (10) and (12) written for the I-th and $(I+1)$-th iteration we easily obtain the formula for the corresponding residual surface gravitational effect $a_{rm,I+1}(x,y)$

$$a_{rm,I+1}(x,y) = a_{rm,I}(x,y) - S(x,y)\,\delta_{m,I+1}(*,*,*), \tag{18}$$

where

$$\delta_{m,I+1}(x,y,z) = \rho_{m,I+1}(x,y,z) - \rho_{m,I}(x,y,z). \tag{19}$$

This formula has the advantage that by the integration on the rhs of (18) we have to consider only the cells whose density has changed.

The above described iterative process is performed until the residual χ-density is, in the whole calculation domain, absolutely smaller than some given value. Another criterion (which does not require to calculate the residual χ-density in the whole calculation domain) is that the residual surface gravitational effect differs from some constant less than some given value. However, the first criterion is better, as there can be anomalous bodies that could not be detected by the calculation, since they are located outside the chosen calculation domain (for example, they are deeper than the maximal depth of calculation).

The iterative calculation of the multi-domain solution of the inverse problem is accomplished without any outer intervention; the only quantities that have to be input are (apart from the parameters of the zero model) the values of the densities of the germs (by the construction of the starting model). However, it does not mean, that it is not possible to change certain parameters between any two iterative steps: if it is desirable, we can introduce new germs or change the density in one or more domains of the current model (of course, the corresponding surface gravitational effect has to be calculated for the changed model). As it was noted before, any such change means that we obtain another final model, but this may be done intentionally, if the current model does not satisfy us. In this way we can improve also the final model without the need of performing the whole calculation from the beginning (of course, we have to perform a sufficient number of iterations with the changed model).

4. Numerical example of the iterative procedure

The numerical calculations of the solution of the inverse gravimetric problem according to the above described procedure were performed for the region of the Kolárovo gravity anomaly and for the region of the eastern part of Slovakia. In both cases the iterative procedure worked well and the evolution of the shapes of the anomalous bodies from the original germs to their final form was shown to be steady.

In the case of the Kolárovo region the calculation domain had dimensions $40 \times 32 \times 16$ km and the step (the length of the edges of the cells) was 0.2

km (thus there were totally 2588880 cells). The zero model consisted of 6 layers with densities from 2500 $\mathrm{kg\,m^{-3}}$ (the upper layer 3 km thick) to 2780 $\mathrm{kg\,m^{-3}}$ (in the depth greater than 15 km); the starting model contained 172 germs of anomalous bodies with densities from 2360 to 3020 $\mathrm{kg\,m^{-3}}$ (the densities of germs were determined from the values of the local extrema of the χ-density). The calculation consisted of 256 iterative steps and the final shape and location of the main anomalous body were similar to the previous calculation (see *Pohánka (2001)*).

In the case of the region of the eastern part of Slovakia the calculation domain had dimensions 200 × 140 × 50 km and the step was 0.5 km (there were totally 11268100 cells). The zero model consisted of 8 layers with densities from 2680 $\mathrm{kg\,m^{-3}}$ (the upper layer 3 km thick) to 3300 $\mathrm{kg\,m^{-3}}$ (in the depth greater than 32 km); the starting model contained 1492 germs of anomalous bodies with densities from 2140 to 3300 $\mathrm{kg\,m^{-3}}$ (the densities of germs were determined as in the previous case). The calculation consisted of 384 iterative steps and the final distribution of anomalous bodies has shown the complicated structure resembling the surface features in the regions just beneath the surface. It has to be noted that the aim of this calculation was to test the method and not to obtain an information about the structure of the earth crust in the particular region; in fact, the planar version of the method used by the calculation could not account for the ellipticity of the earth surface and for the local topography.

In the accompanied figures we present as an example one vertical profile from the calculation domain (length 200 km, depth 50 km). Figure 1 shows at the top the original χ-density and at the bottom the final residual χ-density (after 384 iterations). We see that the domains containing the greatest local extrema of the original χ-density are barely visible after the removal of the gravitational effect of the final model.

Figs 2a – 2e illustrate the growth of the anomalous bodies from the original germs to their final form: the starting model is in Fig. 2a at the top (we can see several germs) and the subsequent iterations are depicted on the succeeding figures. We see that the germs begin to grow in different iterations (this is due to the decreasing limiting value for the residual χ-density); the changes of the boundaries of the main horizontal layers begin also in different iterations. The figures show that the shape of the main inhomogeneities changes minimally near the end of the calculation.

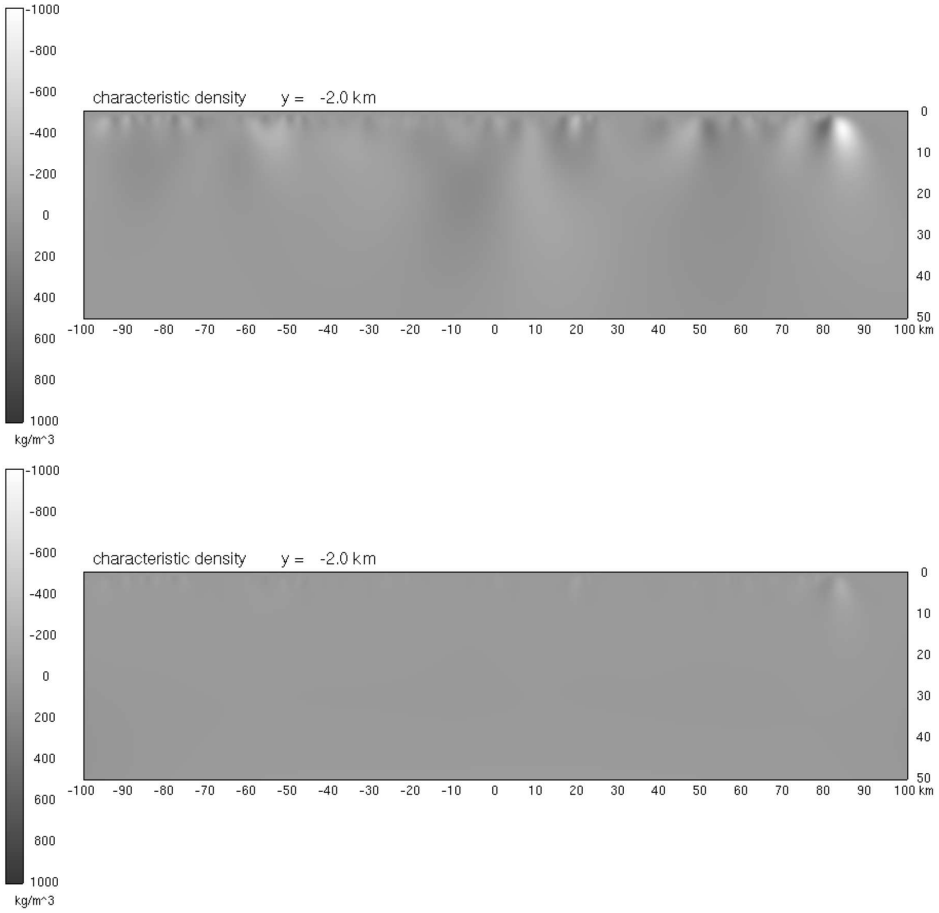

Fig. 1. The original χ-density (top) and the final residual χ-density (bottom) for the chosen vertical profile in the region of the eastern part of Slovakia.

The numerical calculation has also shown the problematical points of the iterative procedure. Perhaps the most important is the so called boundary effect: it appears on the boundary of the calculation domain (mostly on the side boundaries). The substance of the boundary effect is the following: if there is an anomalous body whose part lies outside the calculation domain,

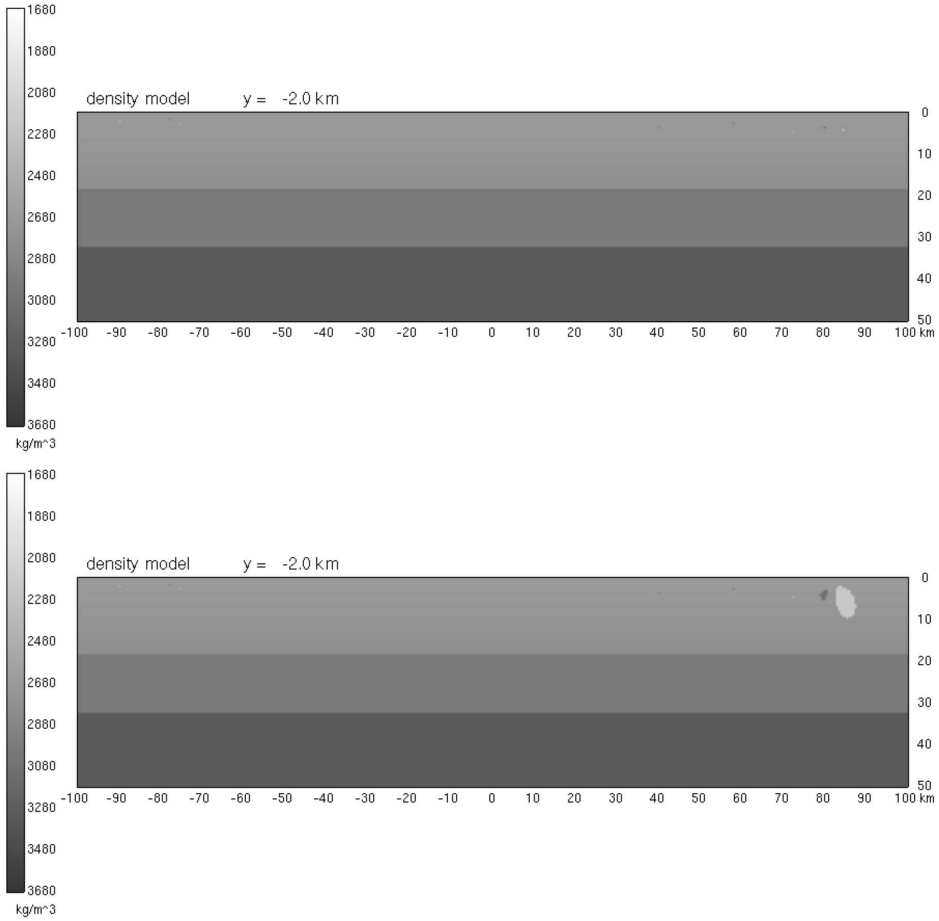

Fig. 2a. The density distribution for the same vertical profile in the 0-th iteration (the starting model, top) and in the 72-nd iteration (bottom).

the surface gravitational effect generated by this body causes the creation of a domain in the model that is responsible for this effect. However, this domain lies entirely within the calculation domain and thus it cannot acquire the form approximating the anomalous body (in order to generate the same surface gravitational effect); this causes that the iterative procedure tends

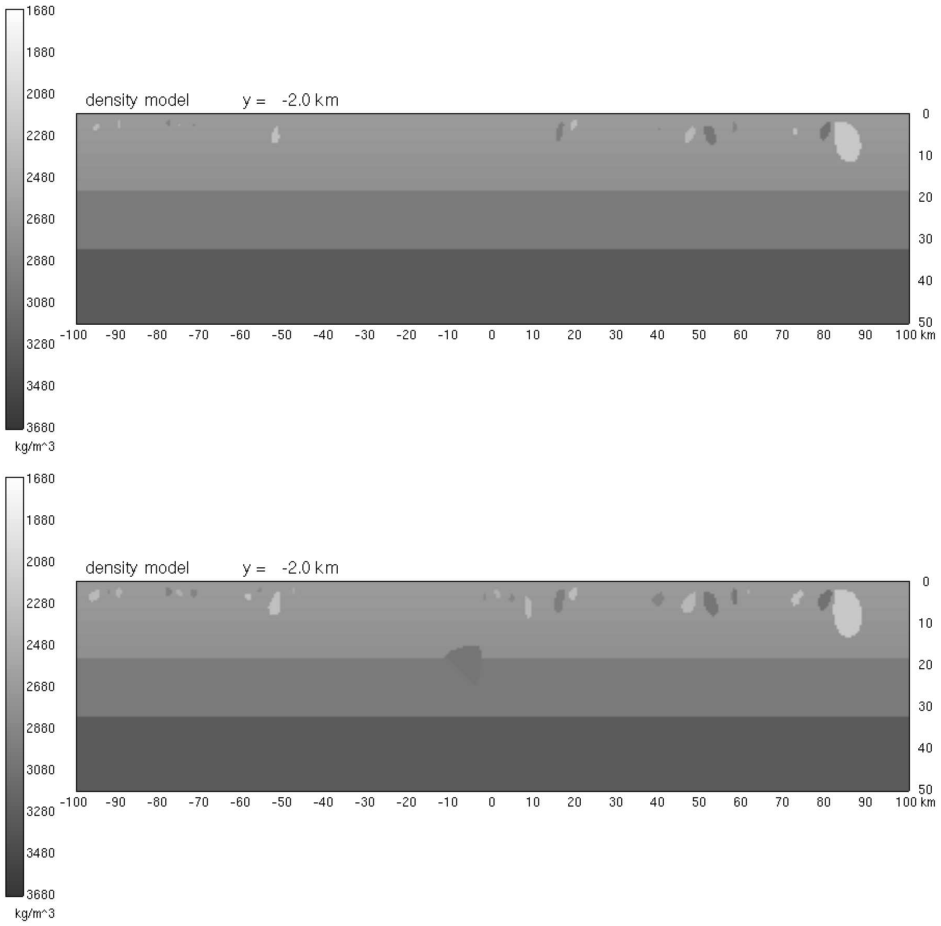

Fig. 2b. The density distribution for the same vertical profile in the 128-th iteration (top) and in the 165-th iteration (bottom).

to expand this domain within the calculation domain and thus the shape of this domain is deformed with respect of the domains lying entirely within the calculation domain.

Unfortunately, the boundary effect cannot be removed by increasing the dimensions of the calculation domain: the effect is removed for the bodies at

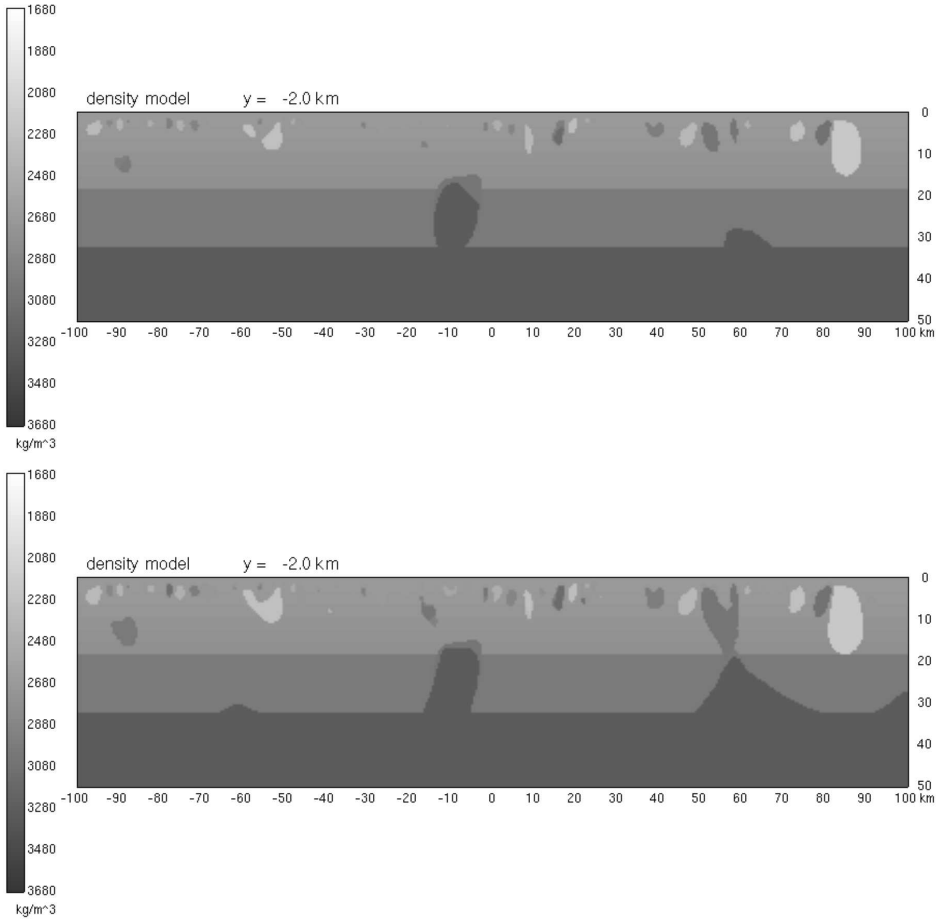

Fig. 2c. The density distribution for the same vertical profile in the 203-rd iteration (top) and in the 256-th iteration (bottom).

the original boundary, but there can be other bodies at the new boundary. This means that the calculation should be performed for a larger calculation domain, but the reliable model should be obtained by restricting the final model to the desired (smaller) calculation domain. The anomalous bodies whose form may be distorted by the boundary effect are easily recognizable

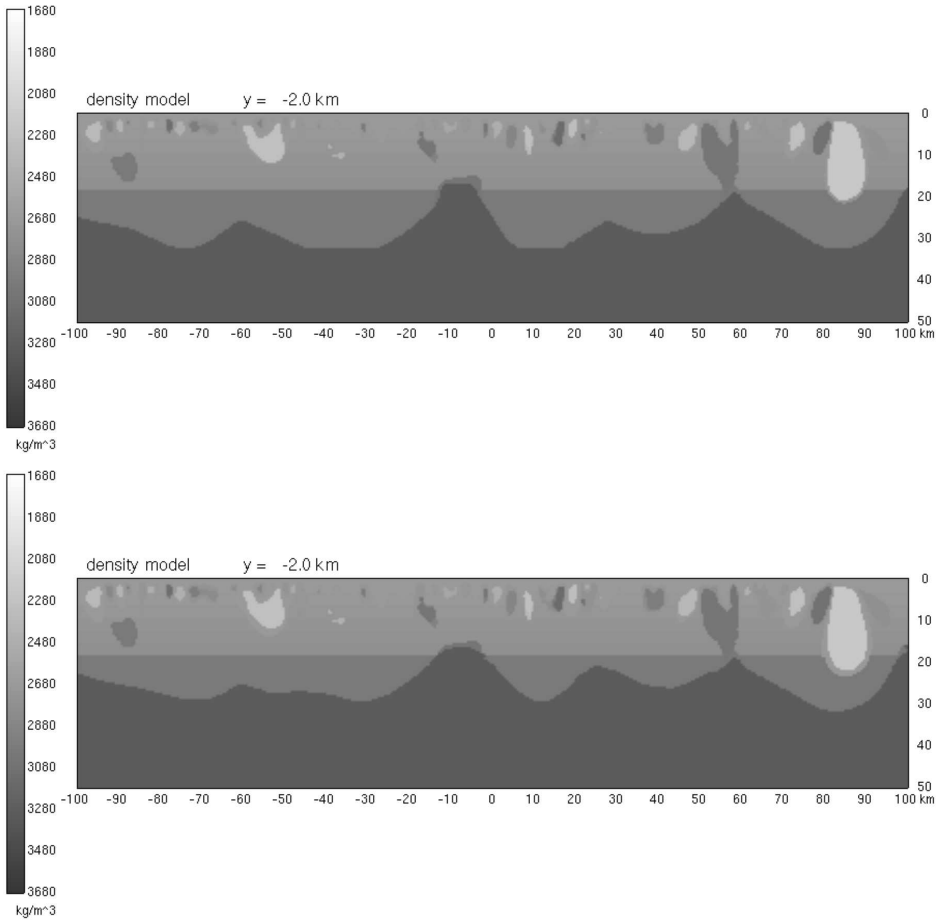

Fig. 2d. The density distribution for the same vertical profile in the 296-th iteration (top) and in the 320-th iteration (bottom).

(at least a part of their boundary lies at the side boundary of the calculation domain) and this allows to determine the necessary amount of cutting of the calculation domain at its sides.

There can also arise a problem with small anomalous bodies lying nearly the earth surface: these bodies may occupy only a few cells and thus their

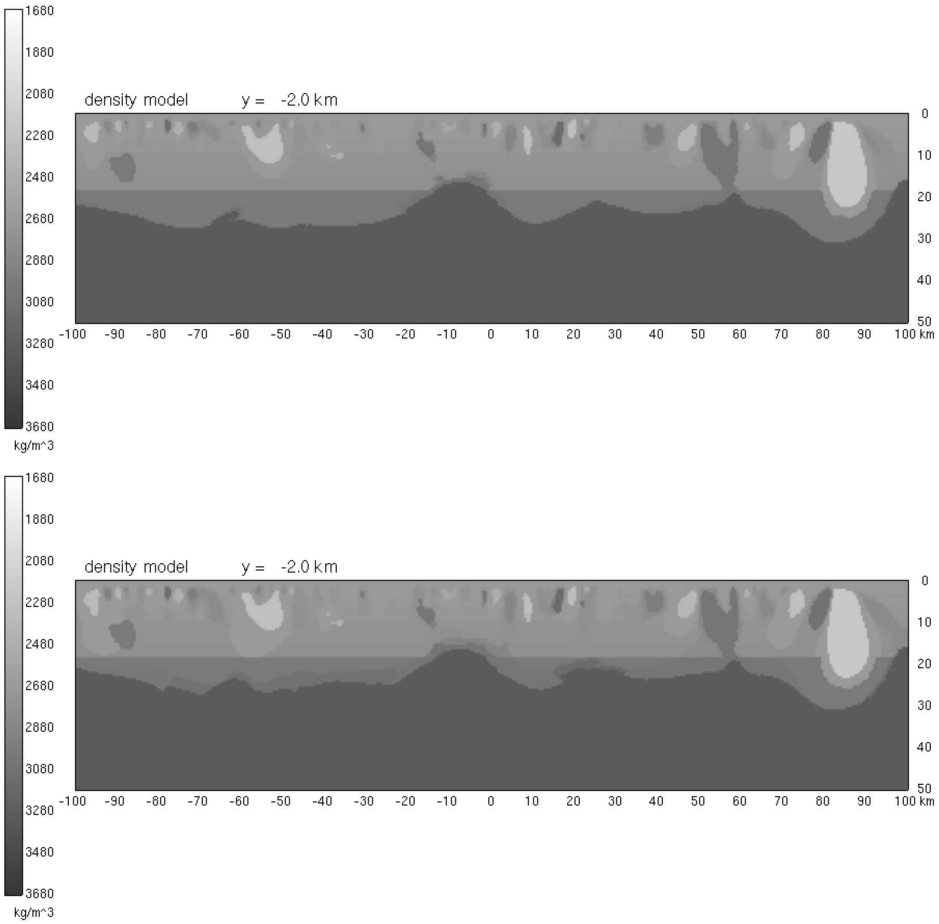

Fig. 2e. The density distribution for the same vertical profile in the 352-nd iteration (top) and in the 384-th iteration (bottom).

form need not be precise enough for the full removing of their surface gravitational effect. This can be repaired either by performing the whole calculation with smaller cells or by suitable smoothing of the measured surface gravitational effect in order to remove the smallest subsurface anomalous bodies.

5. Conclusions

The numerical calculations have shown the correctness of the described iterative procedure: this fact is important as there seems to be no way to prove exactly that the procedure will converge. Further it was shown that small changes of the initial parameters do not lead to a substantially different results, what is remarkable, as the inverse problem of gravimetry is an incorrectly defined problem (there are very different solutions for two nearly identical inputs and even for the single input). The cause for the stability of the solution is that the multi-domain density distribution, which is obtained from the set of germs located only at the local extrema of the original χ-density, is in some sense the simplest possible solution (with the form of a multi-domain density). Introduction of any other germs causes that the resulting model is less simple and the number of possible solutions increases rapidly.

With respect to the future we note that the existing variants of the harmonic inversion method are suitable only for the case of planar earth surface and it is necessary to generalize the method for the real surface (with topography) without the need of any corrections to the gravity values measured on the surface. The iterative procedure presented here can be easily generalized for the case of almost arbitrary surface; the only problem is the calculation of the χ-density (or some other suitable quantity) from the surface gravitational effect measured on the real nonplanar surface.

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References

Pohánka V., 2001: Application of the harmonic inversion method to the Kolárovo gravity anomaly. Contr. Geoph. Geod., **31**, 4, 603–620.