Contr. Geophys. Inst. Slov. Acad. Sci., 27, 1997

Method for the solution of the inverse problem of gravimetry for a planetary body of arbitrary shape

V. Pohánka
Geophysical Institute of the Slovak Academy of Sciences[1]

Abstract: The inverse problem of gravimetry for a planetary body of arbitrary smooth shape is solved by a suitable expression of the gravity potential in the interior of the body, provided the solution of the interior Dirichlet and exterior Neumann problem for the Laplace equation for the domain representing the interior of the body is known. The obtained solution of the inverse problem is general: from this solution it is possible to find any particular solution.

Key words: gravity potential, Laplace equation

1. Introduction

The formulation of the inverse problem problem of gravimetry treated in this paper is as follows: gravity field is assumed to be generated by the matter in the interior of a planetary body of arbitrary shape with a sufficiently smooth surface; at the surface of the body the value of the normal derivative (with respect to the surface) of the gravity potential is given (as input); the problem is to find every density function (from some given class of functions) generating the given external gravity field.

We denote the domain representing the interior (exterior) of the body as D_{int} (D_{ext}) and the surface of the body (the boundary of these domains) as S. Potential of the gravity field $V(\boldsymbol{r})$ and density of the matter $\rho(\boldsymbol{r})$ are defined in the whole space; they satisfy outside the body the equations

Address: [1]Dúbravská cesta 9, 842 28 Bratislava, Slovak Republic

$$\boldsymbol{r}\in D_{\rm ext}: \quad \rho(\boldsymbol{r}) = 0, \tag{1.1}$$

$$\boldsymbol{r}\in D_{\rm ext}: \quad \Delta V(\boldsymbol{r}) = 0 \tag{1.2}$$

and the potential tends to zero at infinity; within the body they satisfy the Poisson equation

$$\boldsymbol{r}\in D_{\rm int}: \quad \Delta V(\boldsymbol{r}) = 4\pi\kappa\,\rho(\boldsymbol{r}), \tag{1.3}$$

where κ is the gravitational constant. As the potential and its gradient are continuous in the whole space, their limits on the surface of the body from inside and outside have to be the same. We shall assume that at any point $\boldsymbol{s}$ of surface S the unit vector of the external normal to this surface $\boldsymbol{n}(\boldsymbol{s})$ is defined, and this vector is a continuous function of $\boldsymbol{s}$. If we denote the inner (outer) limit of function $f(\boldsymbol{r})$ at the point $\boldsymbol{s}$ of surface S as $[f(\boldsymbol{s})]_{\rm int}$ ($[f(\boldsymbol{s})]_{\rm ext}$) and the normal component of the inner (outer) limit of the gradient of this function at the point $\boldsymbol{s}$ as $[\nu_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm int} = \boldsymbol{n}(\boldsymbol{s})\cdot[\nabla_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm int}$ ($[\nu_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm ext} = \boldsymbol{n}(\boldsymbol{s})\cdot[\nabla_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm ext}$), the continuity conditions for the potential at surface S are

$$[V(\boldsymbol{s})]_{\rm int} = [V(\boldsymbol{s})]_{\rm ext}, \tag{1.4}$$

$$[\nu_{\boldsymbol{s}} V(\boldsymbol{s})]_{\rm int} = [\nu_{\boldsymbol{s}} V(\boldsymbol{s})]_{\rm ext}. \tag{1.5}$$

The function on the r.h.s. of the last equation is the input of the inverse problem; this function will be denoted as $g(\boldsymbol{s})$, thus

$$[\nu_{\boldsymbol{s}} V(\boldsymbol{s})]_{\rm ext} = g(\boldsymbol{s}); \tag{1.6}$$

our aim is to find a formula expressing function $\rho(\boldsymbol{r})$ in domain $D_{\rm int}$.

2. Solution of the inverse problem

The idea of solution of the inverse problem is as follows. We express potential $V(\boldsymbol{r})$ in the interior of the body in the form

$$\boldsymbol{r}\in D_{\rm int}: \quad V(\boldsymbol{r}) = U_0(\boldsymbol{r}) + Q(\boldsymbol{r})\,U_1(\boldsymbol{r}) + Q(\boldsymbol{r})^2\,W(\boldsymbol{r}), \tag{2.1}$$

where functions $Q(\boldsymbol{r})$, $U_0(\boldsymbol{r})$, $U_1(\boldsymbol{r})$ and $W(\boldsymbol{r})$ have in domain $D_{\rm int}$ bounded derivatives of the second order and function $Q(\boldsymbol{r})$ satisfies the following conditions:

$$\boldsymbol{r}\in D_{\rm int}: \quad Q(\boldsymbol{r}) > 0, \tag{2.2}$$

$$[Q(\boldsymbol{s})]_{\rm int} = 0, \tag{2.3}$$

$$-\,[\nu_{\boldsymbol{s}} Q(\boldsymbol{s})]_{\rm int} \geq c > 0, \tag{2.4}$$

where c is a suitable constant. It is clear that there is a wide class of functions satisfying conditions (2.2)–(2.4); the problem is rather to find a sufficiently simple example of such a function (this will be treated in a separate paper).

Conditions (2.2)–(2.4) imply that gradient of the function $Q(\boldsymbol{r})$ at any point $\boldsymbol{s}$ of surface S is parallel to the unit vector $\boldsymbol{n}(\boldsymbol{s})$; thus this vector can be expressed in terms of function $Q(\boldsymbol{r})$:

$$\boldsymbol{n}(\boldsymbol{s}) = -\,\frac{1}{K(\boldsymbol{s})}\,[\nabla_{\boldsymbol{s}} Q(\boldsymbol{s})]_{\rm int}, \tag{2.5}$$

where

$$K(\boldsymbol{s}) = |[\nabla_{\boldsymbol{s}} Q(\boldsymbol{s})]_{\rm int}|. \tag{2.6}$$

Then it is clear that

$$[\nu_{\boldsymbol{s}} Q(\boldsymbol{s})]_{\rm int} = -\,K(\boldsymbol{s}) \tag{2.7}$$

and thus according to (2.4) function $1/K(\boldsymbol{s})$ is bounded.

Inserting expression (2.1) in conditions (1.4), (1.5) and using (2.3), (2.7) we get

$$[U_0(\boldsymbol{s})]_{\rm int} = [V(\boldsymbol{s})]_{\rm ext}, \tag{2.8}$$

$$[\nu_{\boldsymbol{s}} U_0(\boldsymbol{s})]_{\rm int} -\, K(\boldsymbol{s})\,[U_1(\boldsymbol{s})]_{\rm int} = [\nu_{\boldsymbol{s}} V(\boldsymbol{s})]_{\rm ext}. \tag{2.9}$$

These equations together with expression (2.1) indicate that it would be advantageous to require that functions $U_0(\boldsymbol{r})$ and $U_1(\boldsymbol{r})$ are harmonic:

$$\boldsymbol{r}\in D_{\rm int}: \quad \Delta U_0(\boldsymbol{r}) = 0, \tag{2.10}$$

$$\boldsymbol{r}\in D_{\rm int}: \quad \Delta U_1(\boldsymbol{r}) = 0. \tag{2.11}$$

The reason is that a function harmonic in some domain is uniquely determined by its value at the boundary of this domain; therefore the amount of information needed to describe such a function completely is smaller than for any other function with the same boundary value. Moreover, according to (1.3) we have then for the density the expression

$$\boldsymbol{r}\in D_{\rm int}: \quad \rho(\boldsymbol{r}) = \frac{1}{4\pi\kappa}\,\Delta\Big(Q(\boldsymbol{r})\,U_1(\boldsymbol{r}) + Q(\boldsymbol{r})^2\,W(\boldsymbol{r})\Big), \tag{2.12}$$

in which function $U_0(\boldsymbol{r})$ does not appear.

Note also that conditions (2.8), (2.9) do not contain function $W(\boldsymbol{r})$ and therefore this function can be chosen arbitrarily (it has only to have in domain $D_{\rm int}$ bounded derivatives of the second order).

Let us now assume that the Green function $G(\boldsymbol{r},\boldsymbol{s})$ for the Dirichlet problem for the Laplace equation in domain $D_{\rm int}$ and the Green function $D(\boldsymbol{r},\boldsymbol{s})$ for the Neumann problem for the Laplace equation in domain $D_{\rm ext}$ are known. Then we have for any function $u(\boldsymbol{r})$ harmonic in $D_{\rm int}$ and such that $[u(\boldsymbol{s})]_{\rm int}$ is sufficiently smooth

$$\boldsymbol{r}\in D_{\rm int}: \quad u(\boldsymbol{r}) = \frac{1}{4\pi}\int_S {\rm d}\sigma\; G(\boldsymbol{r},\boldsymbol{s})\,[u(\boldsymbol{s})]_{\rm int} \tag{2.13}$$

and for any function $v(\boldsymbol{r})$ harmonic in $D_{\rm ext}$ and such that $[\nu_{\boldsymbol{s}} v(\boldsymbol{s})]_{\rm ext}$ is sufficiently smooth

$$\boldsymbol{r}\in D_{\rm ext}: \quad v(\boldsymbol{r}) = \frac{1}{4\pi}\int_S {\rm d}\sigma\; D(\boldsymbol{r},\boldsymbol{s})\,[\nu_{\boldsymbol{s}} v(\boldsymbol{s})]_{\rm ext} \tag{2.14}$$

(${\rm d}\sigma$ is the surface element of surface S at the point $\boldsymbol{s}$).

In the next we shall need an expression for the function $[\nu_{\boldsymbol{s}} u(\boldsymbol{s})]_{\rm int}$, where $u(\boldsymbol{r})$ is given by (2.13). For any function $f(\boldsymbol{r})$ defined in $D_{\rm int}$ and having a sufficiently smooth limit $[f(\boldsymbol{s})]_{\rm int}$, operator $\nabla^S_{\boldsymbol{s}}$ defined as

$$\nabla^S_{\boldsymbol{s}}[f(\boldsymbol{s})]_{\rm int} = [\nabla_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm int} - \boldsymbol{n}(\boldsymbol{s})\;\boldsymbol{n}(\boldsymbol{s})\cdot[\nabla_{\boldsymbol{s}} f(\boldsymbol{s})]_{\rm int} \tag{2.15}$$

represents the tangential part of the gradient with respect to surface S at the point $\boldsymbol{s}$ (note that the l.h.s. of (2.15) can be calculated knowing only the limit value of function $f(\boldsymbol{r})$ on the surface S). Further we define operator $T^S(\boldsymbol{r},\boldsymbol{s})$ (for $\boldsymbol{r}\in D_{\rm int}$) by the formula

$$T^S(\boldsymbol{r},\boldsymbol{s})\,f(*) = f(\boldsymbol{r}) - [f(\boldsymbol{s})]_{\rm int} - (\boldsymbol{r}-\boldsymbol{s})\cdot\nabla^S_{\boldsymbol{s}}[f(\boldsymbol{s})]_{\rm int}; \tag{2.16}$$

it is clear that for a harmonic function $u(\boldsymbol{r})$, function $T^S(\boldsymbol{r},\boldsymbol{s})\,u(*)$ (as a function of $\boldsymbol{r}$) is also harmonic and the normal component of the inner limit of its gradient at the point $\boldsymbol{s}$ is the same as for the function $u(\boldsymbol{r})$. Formula (2.13) for the function $T^S(\boldsymbol{r},\boldsymbol{s})\,u(*)$ reads

$$T^S(\boldsymbol{r},\boldsymbol{s})\,u(*) = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\; G(\boldsymbol{r},\boldsymbol{s}')\,[T^S(\boldsymbol{s}',\boldsymbol{s})\,u(*)]_{\mathrm{int}}, \tag{2.17}$$

where according to (2.16)

$$[T^S(\boldsymbol{s}',\boldsymbol{s})\,u(*)]_{\mathrm{int}} = [u(\boldsymbol{s}')]_{\mathrm{int}} - [u(\boldsymbol{s})]_{\mathrm{int}} - (\boldsymbol{s}'-\boldsymbol{s})\cdot\nabla^S_{\boldsymbol{s}}[u(\boldsymbol{s})]_{\mathrm{int}}; \tag{2.18}$$

then it can be shown that

$$[\nu_{\boldsymbol{s}} u(\boldsymbol{s})]_{\mathrm{int}} = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\; [\nu_{\boldsymbol{s}} G(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{int}}\,[T^S(\boldsymbol{s}',\boldsymbol{s})\,u(*)]_{\mathrm{int}} \tag{2.19}$$

and the integral on the r.h.s. exists, as the integral kernel $[\nu_{\boldsymbol{s}} G(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{int}}$ has a singularity of the type $|\boldsymbol{s}-\boldsymbol{s}'|^{-3}$, while function $[T^S(\boldsymbol{s}',\boldsymbol{s})\,u(*)]_{\mathrm{int}}$ behaves (for sufficiently smooth function $[u(\boldsymbol{s})]_{\mathrm{int}}$) in the neighbourhood of this singularity as $|\boldsymbol{s}-\boldsymbol{s}'|^2$.

In the case of formula (2.14) we can exchange the limit to surface S and integration and we obtain

$$[v(\boldsymbol{s})]_{\mathrm{ext}} = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\; [D(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{ext}}\,[\nu_{\boldsymbol{s}'} v(\boldsymbol{s}')]_{\mathrm{ext}}; \tag{2.20}$$

here the integral kernel $[D(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{ext}}$ is weakly singular.

Now we are able to express the dependence of function $U_1(\boldsymbol{r})$ on the input $g(\boldsymbol{s})$. According to (1.2) we can write formula (2.20) for the function $V(\boldsymbol{r})$; using formulae (1.6) and (2.8) we get

$$[U_0(\boldsymbol{s})]_{\mathrm{int}} = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\; [D(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{ext}}\, g(\boldsymbol{s}') \tag{2.21}$$

and formula (2.19) for the function $U_0(\boldsymbol{r})$ reads

$$[\nu_{\boldsymbol{s}} U_0(\boldsymbol{s})]_{\mathrm{int}} = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\; [\nu_{\boldsymbol{s}} G(\boldsymbol{s},\boldsymbol{s}')]_{\mathrm{int}}\,[T^S(\boldsymbol{s}',\boldsymbol{s})\,U_0(*)]_{\mathrm{int}}. \tag{2.22}$$

From formulae (2.9) and (1.6) we get

$$[U_1(\boldsymbol{s})]_{\mathrm{int}} = \frac{1}{K(\boldsymbol{s})}\Big([\nu_{\boldsymbol{s}} U_0(\boldsymbol{s})]_{\mathrm{int}} - g(\boldsymbol{s})\Big) \tag{2.23}$$

and formula (2.13) for the function $U_1(\boldsymbol{r})$ reads ($\boldsymbol{r}\in D_{\rm int}$)

$$U_1(\boldsymbol{r}) = \frac{1}{4\pi}\int_S \mathrm{d}\sigma\; G(\boldsymbol{r},\boldsymbol{s})\,[U_1(\boldsymbol{s})]_{\rm int}. \tag{2.24}$$

Therefore, the density in the interior of the body is given by (2.12), where function $U_1(\boldsymbol{r})$ is given by (2.21)–(2.24). We see that the density is the sum of two parts: the first part is uniquely determined by the input (the surface gravity field), while the second part depends on the function $W(\boldsymbol{r})$ which can be chosen arbitrarily. Thus this second part represents (in an exact way) the nonuniqueness of the inverse problem of gravimetry. It has to be noted that this second part of density generates the zero exterior gravity field.

3. Solution with extended input

In many cases the surface value of the density (thus function $[\rho(\boldsymbol{s})]_{\rm int}$) is also known. Therefore, it will be suitable to modify the above presented solution of the inverse problem to account for the additional knowledge.

This can be done very simply as follows: we write function $W(\boldsymbol{r})$ in the form

$$\boldsymbol{r}\in D_{\rm int}: \qquad W(\boldsymbol{r}) = U_2(\boldsymbol{r}) + Q(\boldsymbol{r})\,Z(\boldsymbol{r}), \tag{3.1}$$

where functions $U_2(\boldsymbol{r})$ and $Z(\boldsymbol{r})$ have in domain $D_{\rm int}$ bounded derivatives of the second order. Then we get from (2.1)

$$\boldsymbol{r}\in D_{\rm int}: \qquad V(\boldsymbol{r}) = U_0(\boldsymbol{r}) + Q(\boldsymbol{r})\,U_1(\boldsymbol{r}) + Q(\boldsymbol{r})^2\,U_2(\boldsymbol{r}) + Q(\boldsymbol{r})^3\,Z(\boldsymbol{r}) \tag{3.2}$$

and using (2.10) and (2.11) we obtain after some calculation ($\boldsymbol{r}\in D_{\rm int}$)

$$\begin{aligned}\Delta V(\boldsymbol{r}) = {} & (\Delta Q(\boldsymbol{r}))\,U_1(\boldsymbol{r}) + 2\,(\nabla_{\boldsymbol{r}} Q(\boldsymbol{r}))\cdot(\nabla_{\boldsymbol{r}} U_1(\boldsymbol{r})) + \\ & + 2\left[Q(\boldsymbol{r})\,\Delta Q(\boldsymbol{r}) + (\nabla_{\boldsymbol{r}} Q(\boldsymbol{r}))^2\right] U_2(\boldsymbol{r}) + \\ & + 4\,Q(\boldsymbol{r})\,(\nabla_{\boldsymbol{r}} Q(\boldsymbol{r}))\cdot(\nabla_{\boldsymbol{r}} U_2(\boldsymbol{r})) + Q(\boldsymbol{r})^2\,\Delta U_2(\boldsymbol{r}) + \\ & + 3\,Q(\boldsymbol{r})\left[Q(\boldsymbol{r})\,\Delta Q(\boldsymbol{r}) + 2\,(\nabla_{\boldsymbol{r}} Q(\boldsymbol{r}))^2\right] Z(\boldsymbol{r}) + \\ & + 6\,Q(\boldsymbol{r})^2\,(\nabla_{\boldsymbol{r}} Q(\boldsymbol{r}))\cdot(\nabla_{\boldsymbol{r}} Z(\boldsymbol{r})) + Q(\boldsymbol{r})^3\,\Delta Z(\boldsymbol{r}).\end{aligned} \tag{3.3}$$

In the limit to surface S we have according to (2.3) and (2.5)

$$
\begin{aligned}
[\Delta V(\boldsymbol{s})]_{\text{int}} &= [\Delta Q(\boldsymbol{s})]_{\text{int}}\,[U_1(\boldsymbol{s})]_{\text{int}} + 2\,[\nabla_{\boldsymbol{s}} Q(\boldsymbol{s})]_{\text{int}} \cdot [\nabla_{\boldsymbol{s}} U_1(\boldsymbol{s})]_{\text{int}} + \\
&\qquad + 2\,[\nabla_{\boldsymbol{s}} Q(\boldsymbol{s})]^2_{\text{int}}\,[U_2(\boldsymbol{s})]_{\text{int}} = \\
&= L(\boldsymbol{s})\,[U_1(\boldsymbol{s})]_{\text{int}} - 2\,K(\boldsymbol{s})\,[\nu_{\boldsymbol{s}} U_1(\boldsymbol{s})]_{\text{int}} + 2\,K(\boldsymbol{s})^2\,[U_2(\boldsymbol{s})]_{\text{int}},
\end{aligned}
\tag{3.4}
$$

where we denoted

$$
L(\boldsymbol{s}) = [\Delta Q(\boldsymbol{s})]_{\text{int}}. \tag{3.5}
$$

Using formula (1.3) we get the condition

$$
\begin{aligned}
&L(\boldsymbol{s})\,[U_1(\boldsymbol{s})]_{\text{int}} - 2\,K(\boldsymbol{s})\,[\nu_{\boldsymbol{s}} U_1(\boldsymbol{s})]_{\text{int}} + 2\,K(\boldsymbol{s})^2\,[U_2(\boldsymbol{s})]_{\text{int}} = \\
&= 4\pi\kappa\,[\rho(\boldsymbol{s})]_{\text{int}}
\end{aligned}
\tag{3.6}
$$

and we see that we can require that also function $U_2(\boldsymbol{r})$ is harmonic:

$$
\boldsymbol{r}\in D_{\text{int}}: \quad \Delta U_2(\boldsymbol{r}) = 0. \tag{3.7}
$$

Note that condition (3.6) does not contain function $Z(\boldsymbol{r})$ and therefore this function can be chosen arbitrarily (it has only to have in domain D_{int} bounded derivatives of the second order).

Then we can calculate function $U_2(\boldsymbol{r})$ as follows: from (3.6) we have

$$
\begin{aligned}
&[U_2(\boldsymbol{s})]_{\text{int}} = \\
&= \frac{1}{2\,K(\boldsymbol{s})^2}\Big(4\pi\kappa\,[\rho(\boldsymbol{s})]_{\text{int}} - L(\boldsymbol{s})\,[U_1(\boldsymbol{s})]_{\text{int}} + 2\,K(\boldsymbol{s})\,[\nu_{\boldsymbol{s}} U_1(\boldsymbol{s})]_{\text{int}}\Big),
\end{aligned}
\tag{3.8}
$$

where function $[U_1(\boldsymbol{s})]_{\text{int}}$ is given by (2.23), and, in analogy with (2.22)

$$
[\nu_{\boldsymbol{s}} U_1(\boldsymbol{s})]_{\text{int}} = \frac{1}{4\pi}\int_S \mathrm{d}\sigma'\,[\nu_{\boldsymbol{s}} G(\boldsymbol{s},\boldsymbol{s}')]_{\text{int}}\,[T^S(\boldsymbol{s}',\boldsymbol{s})\,U_1(*)]_{\text{int}}. \tag{3.9}
$$

Function $U_2(\boldsymbol{r})$ is then given by ($\boldsymbol{r}\in D_{\text{int}}$)

$$
U_2(\boldsymbol{r}) = \frac{1}{4\pi}\int_S \mathrm{d}\sigma\; G(\boldsymbol{r},\boldsymbol{s})\,[U_2(\boldsymbol{s})]_{\text{int}} \tag{3.10}
$$

and the density is expressed by

$$
\boldsymbol{r}\in D_{\text{int}}: \quad \rho(\boldsymbol{r}) = \frac{1}{4\pi\kappa}\,\Delta\Big(Q(\boldsymbol{r})\,U_1(\boldsymbol{r}) + Q(\boldsymbol{r})^2\,U_2(\boldsymbol{r}) + Q(\boldsymbol{r})^3\,Z(\boldsymbol{r})\Big). \tag{3.11}
$$

Therefore, in this case the density in the interior of the body is given by (3.11), where function $U_1(\boldsymbol{r})$ is given by (2.21)–(2.24) and function $U_2(\boldsymbol{r})$ by (3.8)–(3.10). We see that the density is again the sum of two parts: the first part is uniquely determined by the input (the surface gravity field and the surface value of density), while the second part depends on the function $Z(\boldsymbol{r})$ which can be chosen arbitrarily.

In conclusion, it has to be noted that the presented method for the solution of the inverse problem of gravimetry for a planetary body of arbitrary smooth shape is a generalization of the method treated in [1] for the body of spherical shape. In the latter case function $Q(\boldsymbol{r})$ can have very simple form: if the origin of coordinates is at the centre of the body and the radius of the body is R, domain D_{int} is given by the inequality $\boldsymbol{r}^2 < R^2$ and conditions (2.2)–(2.4) are satisfied by the function $1 - \boldsymbol{r}^2/R^2$. In this case the second and third term on the r.h.s. of (3.2) is a biharmonic and 3-harmonic function, respectively. For arbitrary nonspherical shape of the body, these two terms have not this simple behaviour with respect to the Laplace operator; nevertheless, also in this case any term in the decomposition (3.2) can be considered as simpler than the any of the succeeding terms.

Acknowledgement. The author is grateful to the Slovak Grant agency for science (grant No. 2/1063/94) for the partial support of this work.

Received: 26. 1. 1996 Reviewer: V. Ďurikovič

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