INVERSE PROBLEM OF GRAVIMETRY

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INTRODUCTION

Geophysics is, simply speaking, the physics of the earth, and the physics applied to the earth. It deals with the structure, properties, fields, and behavior of the earth in both spatial and temporal dimensions. Gravimetry, one of the branches of geophysics, is devoted to the observation, study, and interpretation of the earth's gravity field in space and time. Gravimetry shares the study of the gravity field with geodesy, where the field is an inseparable part of positioning and navigation. Gravimetry contributes significantly to geology by aiding the determination of the mass distribution below the earth's surface, which has significant applications in terms of prospecting and exploration for hydrocarbons, mineral deposits, water, etc., as well as to general knowledge of earth's structure. The interpretation of temporal changes of the gravity field helps understanding geodynamic phenomena, such as earthquakes, volcanic and magmatic processes, isostatic rebound, tectonics, etc. Microgravimetric observations can contribute even to archeology, by detecting caves or cavities. In many gravimetric applications the task of the day is solving the inverse problem. Here we shall discuss the recent developments in formulating and solving the gravimetric inverse problem, including investigations in the field of applications, focusing on work carried out in Slovakia.

THEORETICAL BACKGROUND

Our contribution shall be based on the commonly known concepts of the theory of the gravity field, such as gravity and gravitational potential, normal and disturbing potentials, actual and normal gravity, gravity anomaly and disturbance, geoid and reference ellipsoid, etc. (e.g., Kellogg, 1929; MacMillan, 1930; Heiskanen and Moritz, 1967; Bomford, 1971; Vaníček and Krakiwsky, 1986; Blakely, 1995; Torge, 2001).

DIRECT PROBLEM

Having a mass density distribution ρ known inside a body, such as the earth, the actual gravitational potential V (both internal and external), or quantities derived thereof (by applying a differential operator D), are uniquely determined by the Newton's volume integral over the density distribution

$$D\{V(P)\} = G \iiint_{earth} \rho(Q) D\{L^{-1}(P,Q)\} d\vartheta \ , \quad (1)$$

with an integral kernel $K(P,Q) \equiv D\{L^{-1}(P,Q)\}$, where G is the gravitational constant, L^{-1} is the reciprocal Euclidean distance between the evaluation point P and the (dummy) integration point Q, and $d\vartheta$ is infinitesimal volume increment. The task of computing the potential, or other quantities (parameters) of the gravitational field derived thereof, from a known density distribution is referred to as the *direct problem*. This task boils down to the numerical evaluation of the Newton volume integral. It can be done either rigorously in ellipsoidal coordinates, or various approximations are adopted in practice (cf. e.g., Novák and Grafarend, 2005; Vajda et al., 2004b).

INVERSE PROBLEM

The inverse problem is also based on Eq. (1). It is a task of determining the density distribution inside the body from the observed quantity $D\{V(P)\}$ being known on or above the surface bounding the body. The inverse problem is **non-unique** and **ill-posed** (e.g., Menke, 1984; Blakely, 1995). The cause of the non-uniqueness is the fact that there are density distributions within a given body, that generate zero external gravitational potential. Any such distribution can be added to the density distribution generating a given external potential to yield exactly the same external potential. The uniqueness issue can be handled by selecting out of many possible solutions only those, that are geologically meaningful, and by using constraints. The constraints typically come from apriori partial geological knowledge, and/or from independent geophysical or other methods such as magnetic, magnetotelluric, seismic, etc.

INVERSE PROBLEM IN ANOMALOUS QUANTITIES

Instead of working with the *real density distribution* and with the *actual gravitational potential*, or quantities derived thereof, [Eq. (1)], it is desirable to deal with an *anomalous density distribution* and the *disturbing potential* (or quantities derived thereof). The advantage of such an approach dwells in working with small (anomalous) quantities and committing less error when applying certain approximations such as the spherical or planar approximation, or neglecting the deflections of the vertical. The inverse problem can be formulated using anomalous quantities by means of decomposing the density distribution into reference and anomalous components $\rho = \rho_R + \delta\rho$ and the actual potential into normal and disturbing potentials. This inevitably leads to treating the potential of topographic masses separately implying the introduction of topographic corrections to anomalous gravity data. For decades the so called 'Bouguer gravity anomaly' has been used in the gravimetric inversion. Several investigators have been indicating (for a review see e.g., Vajda et al., 2006a) that this quantity deviates systematically from that advocated to be more accurate by the so called 'geophysical indirect effect' (for its definition and evaluation see ibid). Recently Vajda et al. (2006a) have rigorously rederived the formulation of the inverse problem in terms of anomalous gravity quantities. They show that the decomposition of the actual potential results in

$$T(P) - V^{ET}(P) = \delta V(P) \ , \qquad (2)$$

where the left hand side represent the topographically corrected disturbing potential, while T is the disturbing potential, V^{ET} is the gravitational potential of the "topography", i.e., the potential of the reference density distribution between the reference ellipsoid and the surface of the earth, and where on the right hand side we have the gravitational potential of the *anomalous density distribution* contained inside the whole earth, i.e., below the topographic surface. Equation (2) is fundamental in formulating the inverse problem by means of anomalous density distribution. However, the disturbing potential is not observable. By applying such differential operators, that lead to observable quantities defined based on disturbing potential, to Eq. (2)

$$D\{T(P)\} - D\{V^{ET}(P)\} = D\{\delta V(P)\} \qquad (3)$$

we can formulate the inverse problem in observable gravity data. For instance by choosing the operator to be the vertical derivative with respect to the geodetic height (with respect to the inward normal to the reference ellipsoid), $D \equiv -\partial/\partial h$, we arrive at the formulation of the inverse problem by means of the gravity disturbance (ibid)

$$\delta g(P) - a^{ET}(P) = \delta a(P) \ . \qquad (4)$$

The left hand side represents the *topographically corrected gravity disturbance*, where δg is the gravity disturbance, a^{ET} is the attraction of topographic masses being defined with the reference ellipsoid as the lower boundary and with reference density distribution, while on the right hand side we have the *attraction of anomalous density distribution* below the topographic surface, δa. Rewritten explicitly in terms of Newton volume integrals, Eq. (4) reads

$$g(P) - \gamma(P) - G \iiint_{topo} \rho_R(Q) J(P,Q) d\vartheta = \\ = G \iiint_{earth} \delta\rho(Q) J(P,Q) d\vartheta \ . \qquad (5)$$

The left hand side of Eq. (5) can be compiled from observed data. It only requires the measurement of the actual gravity g at the observation point P, the evaluation of normal gravity γ at P using analytical formulae (e.g., Somigliana, 1929; Heiskanen and Moritz, 1967; Vajda and Pánisová, 2005), and the numerical evaluation of the topographic correction given by the Newton volume integral over the reference density distribution (which is typically chosen as globally constant density), enclosed by the reference ellipsoid and the earth surface, with the J kernel being the vertical derivative of the reciprocal Euclidean distance, which requires the global knowledge of the topographic surface in terms of ellipsoidal (geodetic) heights. The right hand side of Eq. (5) is a functional of the unknown and sought anomalous density distribution, mediated via a Newton volume integral with the J kernel. Now the task is: Given the known (compiled from observed data) *topographically corrected gravity disturbances* (given on or above the topographic surface), determine the *anomalous density distribution* inside the earth, i.e., below the topographic surface. The strategies for solving such an inverse problem will be discussed in the next section.

TECHNIQUES FOR SOLVING INVERSE PROBLEM

The attempt to solve the inverse problem is in gravimetry called also *gravity data interpretation*. The reason is that we try either to determine the density distribution, or to estimate some of its parameters. The techniques for solving the inverse problem can be divided into three categories (e.g., Parker, 1977; Menke, 1984; Blakely, 1995):

Forward modeling – A starting model of the density distribution is constructed based either on available geologic, geophysical and/or other independent information, or on the intuition of the interpreter. The direct problem is solved and the computed (model, synthetic) gravity data are compared to the observed ones. Based on the mismatch the model is modified (tuned) and the procedure is repeated until a satisfactory match is reached. The modification of the model may be manual, automatic, or semiautomatic.

Although this method is used routinely worldwide, there is room for further research and development. Developments in Slovakia take place in (A) improving or inventing the analytical formulae for computing the gravity for certain classes of models or bodies (e.g., Pohánka, [1]), (B) designing forward modeling software, especially 3D suites, such as 'Mod3D' developed by Igor Cerovský [2], (C) integrating the gravity data with additional geophysical, geologic and/or other information in the process of modeling, e.g., research carried out by Miroslav Bielik and Jana Dérerová (Bielik et al., 2002; 2005; Zeyen et al., 2002; Dérerová et al., 2005a; 2005b), (D) defining procedures for compiling more accurate input gravity data for the forward modeling in particular, and for the inversion in general (Vajda et al., 2006a; 2006b; Vajda and Pánisová, 2005; 2006).

Inversion – The Newton volume integral over the density distribution is parameterized, which requires simplifying assumptions, and the parameters are computed directly from observed data. This problem is linear in density and non-linear in the geometry of the model. To handle the ill condition of the problem, additional constraints may be adopted, such as a most compact source requirement (e.g., Last and Kubik, 1983; Cerovský, 2005).

There is still a lot of room for further research in this area. As examples of the theoretical developments taking place in Slovakia we would like to mention the work carried out by Igor Cerovský (2005), and a methodology, called the 'harmonic inversion method', under development at the Department of Gravimetry and Geodynamics [3] of the Geophysical Institute [4], Slovak Academy of Sciences by Vladimír Pohánka (cf. [1]).

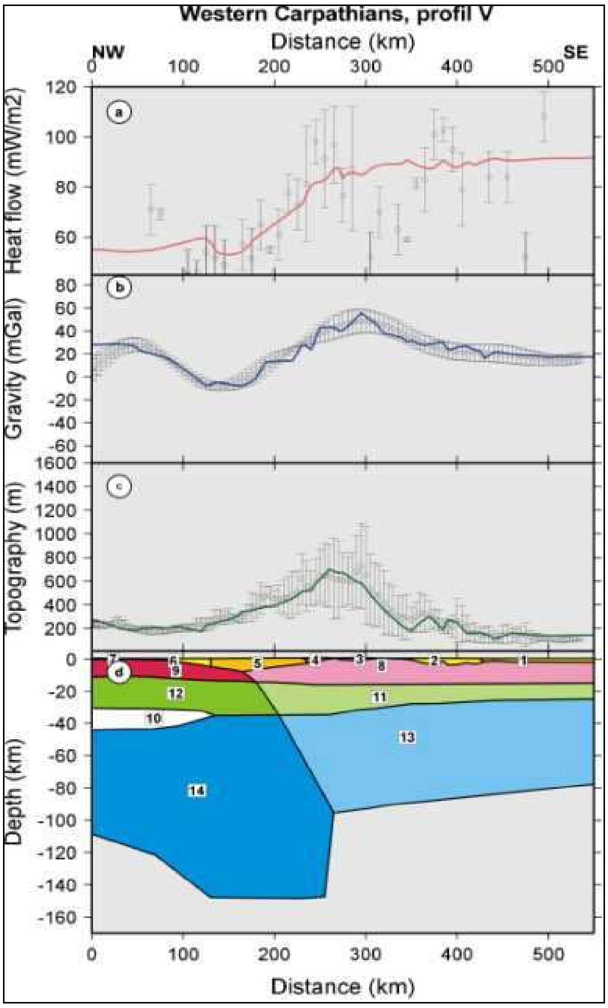

Fig. 1. Integrated modeling of gravity data.

Data enhancement and pattern recognition – Observed gravity data are filtered, transformed, or otherwise processed in order to amplify certain features, which then may be recognized as signatures of certain known geological sources or their elements. Intuition and experience of the interpreter enter the process of interpretation. The association between geological sources and the 'patterns' must be established apriori by means of synthetic modeling and case studies.

The door is wide open for further research in this area. As an example of such developments in Slovakia we would like to mention the development of the so called 'Truncation Filtering Methodology' (TFM) by Peter Vajda and Jaroslava Pánisová. The TFM is a novice technique (Vajda, 1995; Vajda and Vaníček, 1997; 1999) based on transforming gravity data using integral transforms with specific kernels and one free parameter, and producing animated sequences of filtered data in which dynamic patterns are observed and identified [5]. Research is in progress to establish the association between the TFM patterns and the geologic sources.

A special case of inversion is estimating a particular parameter, such as the depth to a source or to a set of certain sources, two techniques being recognized in the practice, namely the *Euler* and *Werner* ***deconvolutions*** (e.g. Blakely, 1995). In Slovakia research is carried out in developing the deconvolution methods by Roman Pašteka (Pašteka and Richter, 2005).

The interpretation (solution of the inverse problem) is inherently non-unique, but one or several admissibly realistic solutions, or classes of solutions may be found. It is very important to employ all available independent information in the interpretive process, such as the knowledge of the geologic and tectonic setting, seismic reflection and/or refraction surveys, previous potential field studies, heat flow, boreholes in the area, etc.

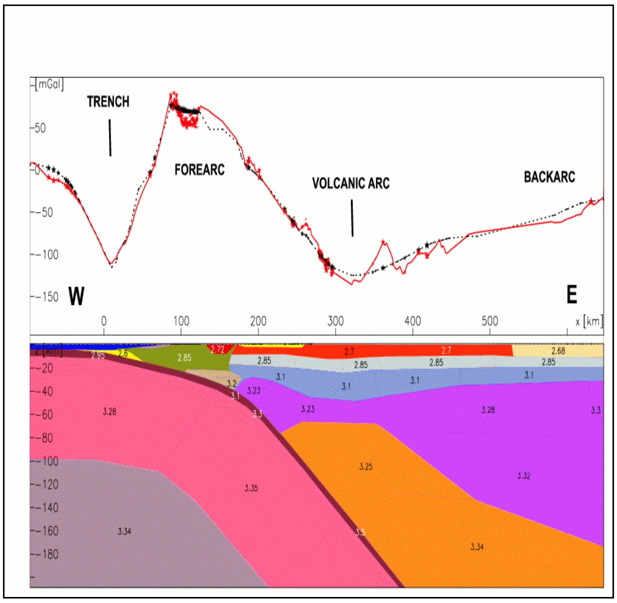

Fig. 2. Forward modeling by means of the 3D modeling software IGMAS, using constraints, of gravity data of a South America's Andes cross-section.

MEASUREMENTS

Gravity measurements can be divided into *absolute* and *relative*. Absolute gravimeters, typically based on the free-fall principle, measure the value of gravity. Currently the best achievable accuracy in absolute gravity is indoors 2 μGal (1 μGal = $10^{-8} ms^{-2}$), by e.g. the FG5 gravimeter [6], and 10 μGal in the field, by e.g., the A10 gravimeter [6]. Relative gravimeters, typically based on elasticity principles, measure the difference of gravity between two stations. Currently the best achievable accuracy in relative gravity is 1 to 5 μGal, by e.g., the CG5 gravimeter [7]. Gravity observations can be performed on land (terrestrial surveys), on sea surface or at sea bottom (ship-borne marine surveys), the meter can be carried onboard a helicopter or a plane (air-borne surveys), taking into account on-flight accelerations. Gravity can be compiled also by means of observations to/by satellites (satellite missions). earth's gravitational field can be determined by analyzing the orbits of earth-orbiting satellites. The determination of the geoid on seas by measurements of the distance from a satellite to the sea surface is known as *satellite altimetry* [8]. New generation of low-orbiting satellites, equipped with highly precise inter-satellite and accelerometry-instrumentation observes the earth gravitational field and its temporal variability at high resolution. Missions CHAMP [9] and GRACE [10] collect and process precise orbits and produce precise monthly global gravity field solutions, thus enabling the study of temporal global gravity field changes. The upcoming European mission, GOCE (ESA, [11]), equipped in addition with a gradiometer, is expected to achieve an accuracy of 1 to 2 cm in the geoid determination with a resolution of 100 km.

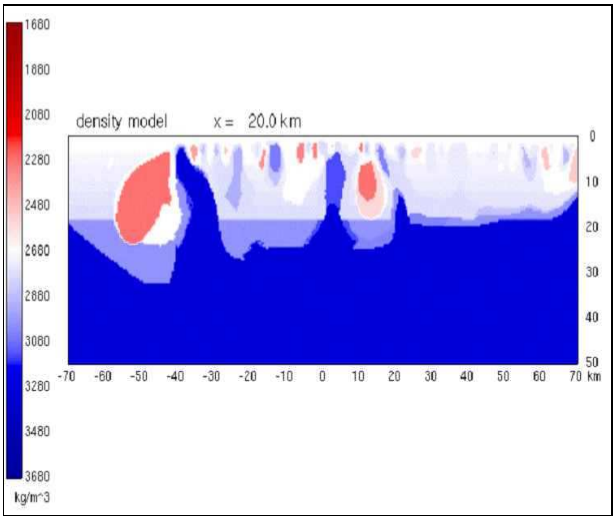

Fig. 3. Interpretation of gravity data of Eastern Slovakia using the 'harmonic inversion method'.

APPLICATIONS

In this section we will show a few examples of how achievements of theoretical developments in the field of gravimetric inversion can be applied in practice, focusing on research carried out in Slovakia.

In the area of *forward modeling* we show in Fig. 1 an example of interpreting the gravity data in the 'Pancardi' region (Pannonian Basin, Carpathians, Dinarides) within the frame of the EUROPROBE's Pancardi project [12], in order to study the lithosphere and asthenosphere by integrated modeling. In the frame of the multilateral project CELEBRATION 2000 (Central European Lithospheric Experiment Based on Refraction), the interpretation of the seismic cross-sections along transects of the project CEL01, CEL04 and CEL05 was initiated, and density modeling along these transects was performed (Bielik et al., 2005b; Dérerová et al., 2005b; [13]). Figure 2 shows gravity data interpretation performed by Zuzana Tašárová (2004), using additional constraining information, for a cross-section in the Andes, by means of the IGMAS 3D modeling software.

In the area of *direct inversion* the 'harmonic inversion method' (Pohánka, [1]) was applied to the gravity data of Eastern Slovakia, cf. Fig. 3 to test the merits of the method.

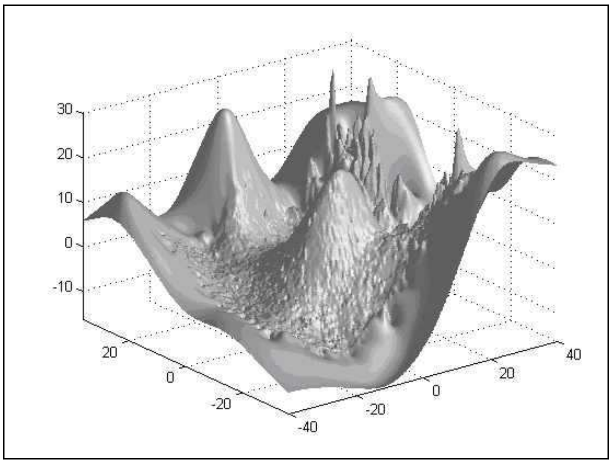

Fig. 4. Interpretation of the Kolárovo gravity high using the 'TFM' methodology.

In the area of *pattern recognition* the TFM methodology was applied to interpret the Kolárovo gravity high in southern Slovakia (Vajda et al., 2002), cf. Fig. 4.

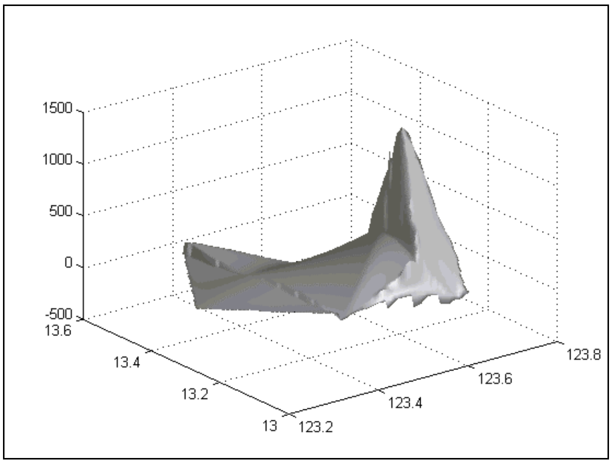

Fig. 5. Interpretation of temporal gravity changes at the Mayon volcano, Philippines, using the 'TFM'.

The observed temporal changes of gravity may be used for studying magmatic and volcanic processes, this subject belonging to the area of geodynamics. Active volcanoes still pose a threat to human lives. Monitoring the volcanic activity and prediction of eruptions thus plays an important role in natural hazards studies. Studying the temporal gravity changes together with surface deformations is an integral part of volcanology and geodynamics. The Department of Gravimetry and

Geodynamics of the Geophysical Institute takes part in research devoted to interpreting temporal gravity changes, with international cooperation (Vajda and Brimich, 2001; Brimich et al., 2002; Vajda et al., 2004a). In two case studies, on the Mayon volcano, Philippines, cf. Fig. 5, and the Merapi volcano, Java, Indonesia, an attempt was made to interpret temporal gravity changes to determine the depth to the magma sources triggering eruptions (Vajda et al., 2004a).

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