Application of the harmonic inversion method to the Kolárovo gravity anomaly

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A b s t r a c t : The method of gravity inversion developed by the author is applied to the Kolárovo gravity anomaly region. After interpolating the original data into a regular grid the characteristic density is calculated. The method for finding the shape of the anomalous body from the characteristic density is developed and applied. The results are the shapes of the anomalous body for four values of density within the body.

Key words: integral transformation, iteration

1. Introduction

The aim of this work was to test the method for the solution of the inverse problem of gravimetry presented in *Pohánka (1998)*. As this simplest variant of the inverse problem treats the surface of the earth as a plane, it was inevitable to choose some relatively small area with negligible topography. Further, as this was to be the first numerical test of the method, the area should contain a single prominent gravity anomaly. Therefore the region of the Kolárovo gravity anomaly was chosen: the input data were the directly measured values of the gravity in this region (author is indebted to Geocomplex Co. for providing the data).

2. Calculation of the characteristic density

The origin of rectangular coordinates was chosen at the point located at $47^\circ\ 57'$ N, $18^\circ\ 00'$ E, height 110 m over the sea level. The plane replacing the

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earth surface by the calculation was orthogonal to the local vertical at the origin (x and y axes are oriented to the local east and north, respectively, z axis is oriented upwards).

The gravity data (totally 8773 measurement points) contained for each point its geographic coordinates, its height above the sea level and the measured absolute value of the acceleration. However, the input of the inversion method mentioned above is the quantity equal to minus vertical component of the gravitational acceleration at the surface (in the next this quantity will be called shortly the surface gravitational effect). Therefore, the data were first corrected for the vertical component of the centrifugal force (it was assumed that the vector of gravity acceleration is parallel to the local vertical) and reduced to the height of the origin (the differences in height were in the most cases only few metres). There was not made the correction for the mass between the earth surface and the height of the origin, because it is intended to generalize the inverse method in the future for the case of the arbitrary form of earth surface (in this case all mass below the earth surface will be considered automatically). Moreover, it has to be stressed that the primary goal of this work was to test the method; the requirement that the solution should correspond to reality was the secondary one.

The measurement points were then projected to the horizontal plane replacing the earth surface; the resulting data set contained for each point its x and y coordinates and the surface gravitational effect. All points were contained in the rectangle 56 km $\times$ 48 km centered at the origin; however, the density of points (in the mean 3.26 points per km^2) was not uniform and there were areas containing no point (one such area with diameter about 2 km was directly near the centre of gravity anomaly).

Therefore, it was necessary to create a regular net of data points (which would be the input for the subsequent calculation); this was done by interpolating the original data using a formula developed by the author. This interpolation formula gives the interpolated value calculated from the given (measured) values at all measurement points; each measurement point contributes to the interpolated value with certain weight decreasing with the distance between the calculation and measurement points. This interpolation formula has also the property that the calculation point can lie well outside the region containing the measurement points (thus the formula is able to perform also the extrapolation); the extrapolated values do not diverge with increasing distance from region of the measurement points. Moreover, the interpolation formula can perform the smoothing of the data according to a given parameter (the smoothing distance).

As in our case the mean minimal distance between the original data points was 0.425 km, the smoothing distance was chosen to be 0.4 km (thus there was a minimal smoothing). In this way it was created a regular net of points in the rectangle 80 km $\times$ 72 km centered at origin with the step 0.2 km for both axes (totally 144761 points). The result is presented in Fig. 1. (here it is already subtracted the mean value of the surface gravitational effect from the whole rectangle); the main gravity anomaly is clearly seen near the centre.

Now it was possible to calculate the characteristic density (or shortly χ-density) for the Kolárovo region. We used the formula (24) of *Pohánka (1998)*, where we have omitted the first term on the rhs containing the surface value of density, as this was the first test of the method (moreover, the data on density were not available). The second term on the rhs of this formula was further modified (by a simple transformation of the integration variable) and the resulting formula for the χ-density now reads (for $z \leq 0$)

$$\chi(x,y,z) = \frac{20}{\pi\,\kappa} \int_0^\infty \mathrm{d}u \, \frac{u^4\,z^4}{(u^2+z^2)^{7/2}} \, \partial_u \frac{1}{u} \, \partial_u \, \bar{a}(x,y,u), \tag{1}$$

where

$$\bar{a}(x,y,u) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \; a(x+u\,\cos\varphi, y+u\,\sin\varphi) \tag{2}$$

and $a(x,y)$ is the surface gravitational effect (equal to minus vertical component of the gravitational acceleration at the surface).

The calculation of χ-density was performed for the points in the rectangular prism the upper boundary of which was the rectangle 40 km $\times$ 32 km centered at origin (with the step 0.2 km for both axes) and the depth of the lower boundary was 10 km (with the step in depth again 0.2 km). As the χ-density is identically zero at the surface, calculation was done for 50 values of depth, thus there were totally 1618050 calculation points.

The reason for the smaller horizontal dimensions of the domain of calculation points with respect to the domain of input points is that the numerical

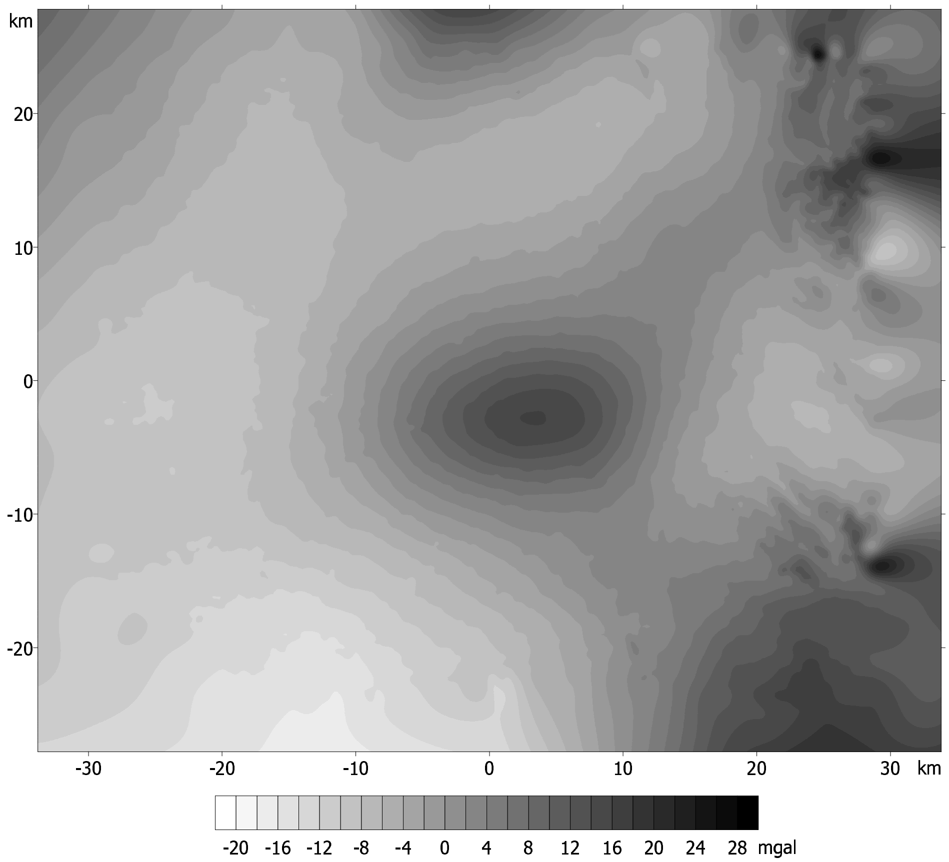

Fig. 1. The surface gravitational effect (equal to minus vertical component of the surface gravitatioal acceleration) around the Kolárovo anomaly after interpolation and subtraction of its mean value (horizontal axis x, vertical axis y).

integration requires that the input points are from a circle (centered above the calculation point) whose radius is at least 5-times the depth of the calculation point. In our case this radius was chosen to be 20 km, thus it was sufficient for the depths to 4 km; nevertheless, as there was no more surface data, we have calculated for the depths to 10 km (to test what happens).

For our input data, the values of acceleration (after subtracting the mean value of the whole data set) were between -20.1 and 26.4 mgal (note that for constant surface gravitational effect the formulae (1), (2) for χ-density give zero resulting value).

The calculation was performed on a computer with Pentium 533 MHz processor; the interpolation of the original data lasted 31 min, while the calculation of the χ-density lasted 42 min.

The calculated χ-density shows clearly the main density inhomogeneity (see Figs. 2, 4, 6, 8): the maximum value of χ-density within this inhomogeneity is 249 kg.m^{-3} and is located at the point with depth 10 km and horizontal coordinates (3.4 km, -2.8 km). This indicates that the true maximum of χ-density can lie still deeper.

3. Calculation of the shape of the anomalous body

Although the χ-density clearly shows the location of the main anomalous body, this result cannot satisfy us, as we are not able to find directly from this information the shape of the body. This is because the χ-density is a smooth function of coordinates, while the realistic distribution of density below the surface should be rather a partially constant function; in other words, the space below the surface is divided into several domains such that the density within each domain is constant. In our case it would correspond (in the first approximation) to a single body whose shape has to be found.

In the following we shall describe the method for calculating the shape of the anomalous body from the χ-density. Consider any density distribution $\rho(x,y,z)$ $(z \leq 0)$; to this function there corresponds the surface gravitational effect $a(x,y)$ described by the well known formula

$$a(x,y) = -\,\kappa \int_{z' \leq 0} \mathrm{d}V' \, \frac{z'}{((x-x')^2 + (y-y')^2 + z'^2)^{3/2}} \, \rho(x',y',z') \tag{3}$$

and the χ-density $\chi(x,y,z)$ given by the formulae (1), (2). This χ-density contains the same amount of information as the surface gravitational effect $a(x,y)$, although the former is a function of 3 variables, while the latter a function of 2 variables. The main difference between these functions is the folowing: as the contributions of particular bodies are in both cases added, by the surface gravitational effect these contributions can overlap (and thus need not be distinguishable), while by the χ-density these are well separated (for well separated bodies). This difference is substantial for example in the case of several bodies located (horizontally) at the same place but with different depths.

Accordingly, for two gravitationally equivalent distributions of density, their surface gravitational effects and their χ-densities are equal. The converse is not always true, as the surface gravitational effect generated by any number of horizontal layers with density constant within each layer is a constant function, and thus the χ-density for such layers is always identically zero. This has the consequence that such horizontal layers are not detectable in the χ-density.

It is important to realize that the density which appears in formula (3) has to decrease with increasing depth in such a way that the integral on the rhs exists; in the case of partially constant density it means that the density has to be zero below some depth. This has the consequence that the density in formula (3) has to be the difference density; this density can be obtained from the real density distribution by subtracting some suitable background density distribution which generates constant surface gravitational effect. In our case it is reasonable to choose as this background just the set of horizontal layers with density constant within each layer. The parameters of these layers (depths and values of density) have to be known a priori, thus they have to be obtained by some different method.

Therefore, in the case of a single anomalous body with constant density located in this horizontally layered background, the distribution of difference density (which appears in formula (3)) is zero outside the body, while within the body it is the difference of the density of the body and the density of the horizontal layer at the particular depth (in other words, body with the constant density will appear as a layered body with different densities within each layer).

The method of finding the shape of the body consists of comparing the original χ-density with the χ-density corresponding to the model body. Recall that the χ-density as a function of the surface gravitational effect is the maximally smooth density distribution creating this surface gravitational effect and satisfying the following conditions:

a. χ-density depends linearly on the surface gravitational effect;

b. for the surface gravitational effect generated by a point source lying

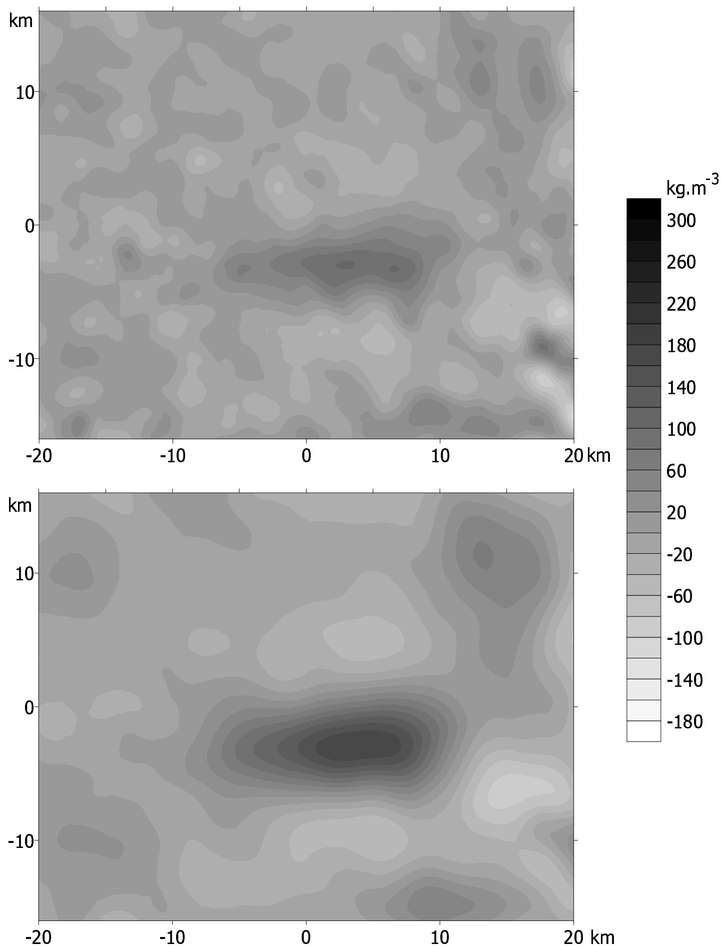

Fig. 2. Calculated χ-density for z = -3 km and z = -5 km (horizontal axis x, vertical axis y).

below the surface, the χ-density has its main extremum at the point source.

As a consequence, for an anomalous body whose horizontal dimensions are not much greater than its vertical dimension, the corresponding χ-density is substantially different from zero in the domain around the body (but prolonged vertically below the body); the sign of the χ-density in this

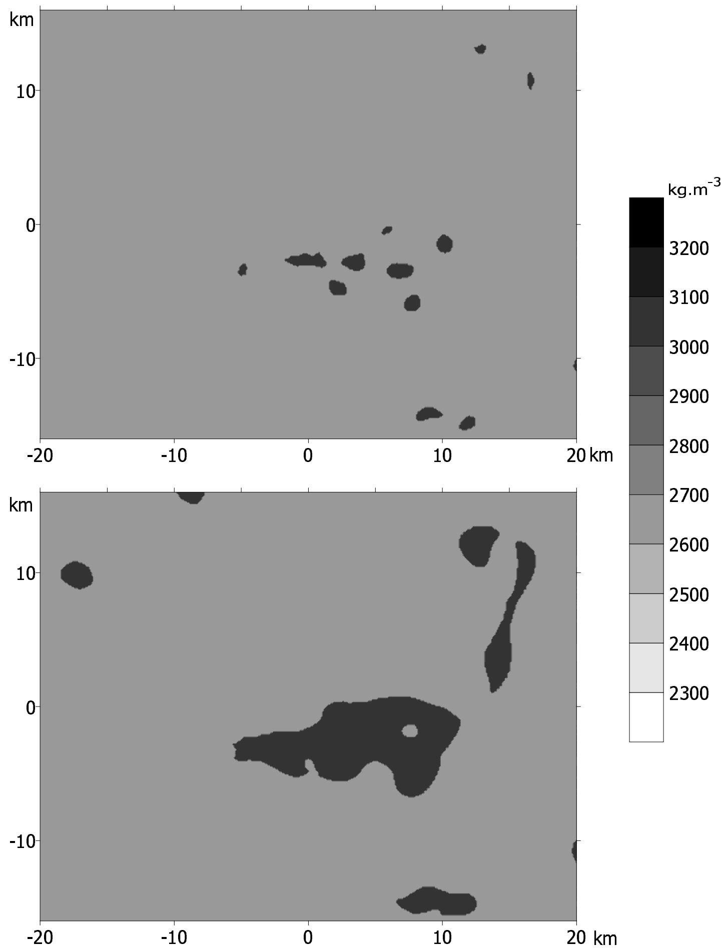

Fig. 3. Calculated density distribution for model body with density 3000 kg.m^{-3} for z = -3 km and z = -5 km (horizontal axis x, vertical axis y).

domain is the same as the sign of the difference density of the body.

For finding the shape of the anomalous body, the only free parameter is the density within the body; this is because there can be gravitationally equivalent bodies with different density and also different shape (the body with higher density is contained within the body with lower density).

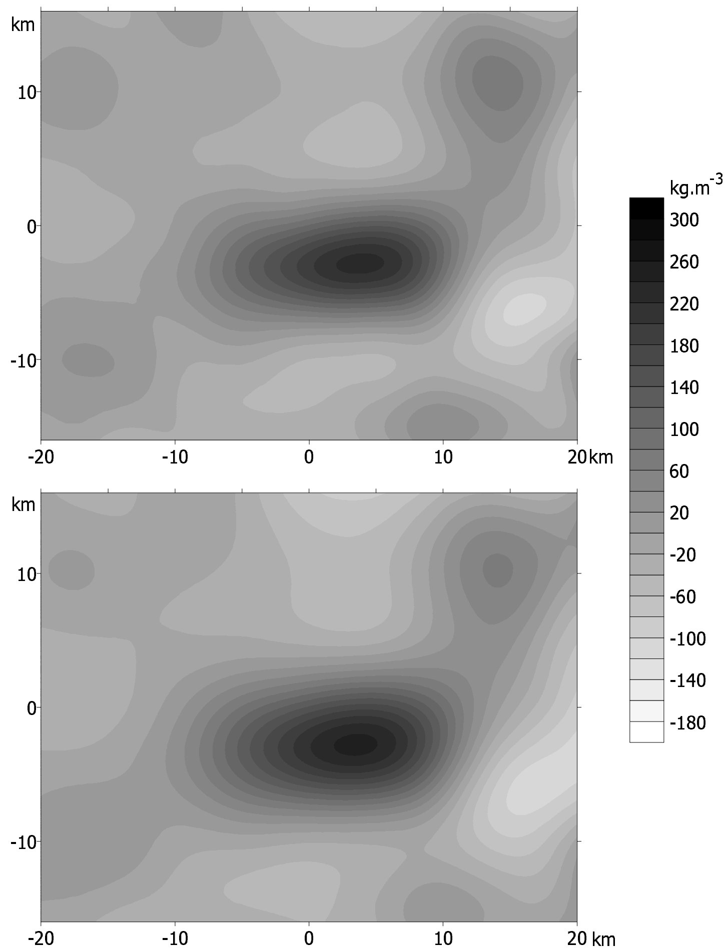

Fig. 4. Calculated χ-density for z = -7 km and z = -9 km (horizontal axis x, vertical axis y).

The shape of the anomalous body can be represented by the depth of the upper and lower boundary of the body within some region in the (x, y) plane. There can be points in this region, where these depths are equal, thus there the body is not present; nevertheless, it is important to have potentially the body also in these points, as by the method described below we can move

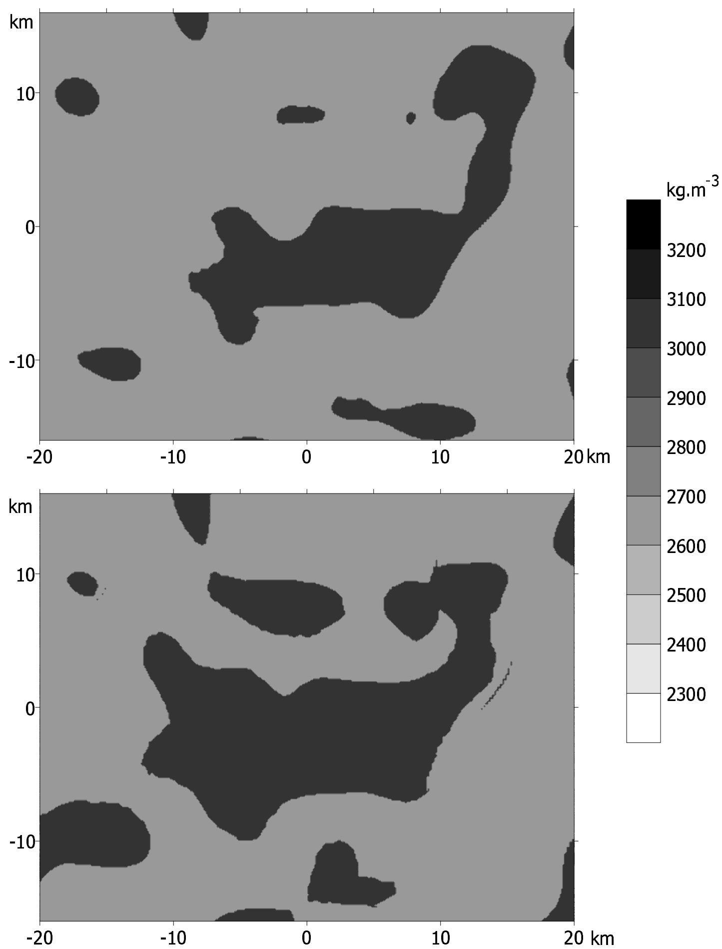

Fig. 5. Calculated density distribution for model body with density 3000 kg.m^{-3} for z = -7 km and z = -9 km (horizontal axis x, vertical axis y).

the upper and lower boundary only up or down, but not horizontally.

Once the density and the boundaries of the body are given (provided the background consisting of the horizontal layers with constant density is also given), we can calculate from the difference density for the model body $\rho_m(x, y, z)$ according to formula (3) the surface gravitational effect

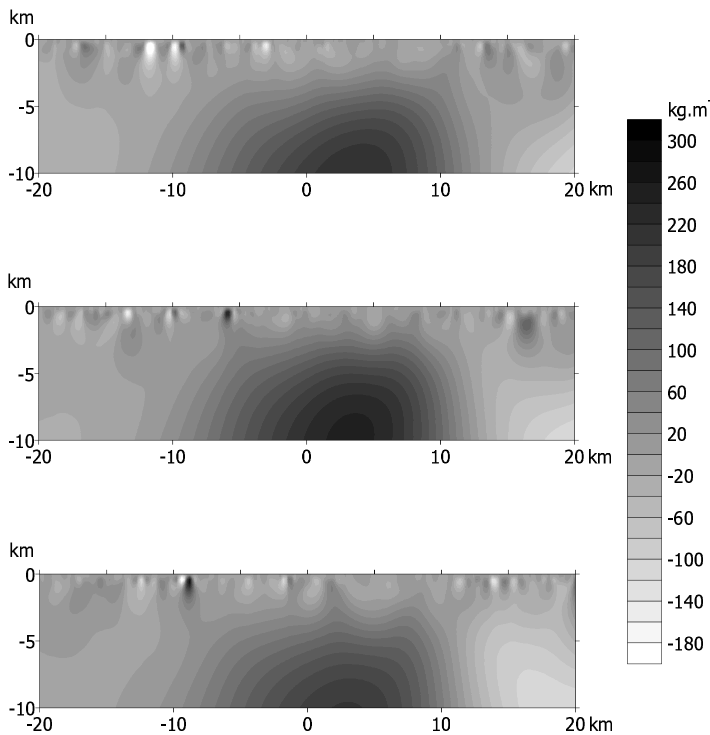

Fig. 6. Calculated χ-density for y = -1 km, y = -3 km and y = -5 km (horizontal axis x, vertical axis z).

of the body $a_m(x,y)$ and according to formulae (1) and (2) the χ-density corresponding to this body $\chi_m(x,y,z)$.

Then we compare the χ-density of the body $\chi_m(x,y,z)$ with the original χ-density $\chi(x,y,z)$ at the points within the body and its near neighbourhood. However, according to the smoothness of the χ-density it is sufficient to compare these two χ-densities at the boundaries of the body. The following procedure is for simplicity described for the case that the difference density within the body is everywhere positive. At any point of the upper boundary (x,y,z), if $\chi_m(x,y,z) < \chi(x,y,z)$, we shift the upper boundary at this point upwards (in order to enlarge the body); if $\chi_m(x,y,z) > \chi(x,y,z)$, we shift the upper boundary downwards (in order to lessen the body). Simi-

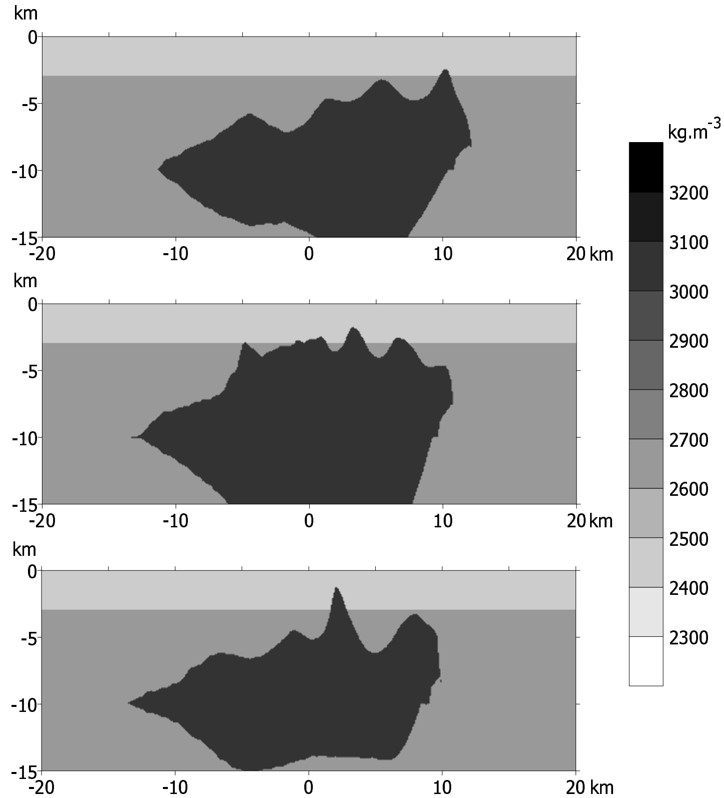

Fig. 7. Calculated density distribution for model body with density 3000 kg.m^{-3} for y = -1 km, y = -3 km and y = -5 km (horizontal axis x, vertical axis z).

larly, at any point of the lower boundary, if $\chi_m(x, y, z) < \chi(x, y, z)$, we shift the lower boundary at this point downwards (in order to enlarge the body); if $\chi_m(x, y, z) > \chi(x, y, z)$, we shift the lower boundary upwards (in order to lessen the body). The amount of shifting can be made proportional to the (relative) difference of the two χ-densities.

In this way we find for the given shape of the body its new shape (whose χ-density should match the original χ-density better than the old one). By this process it can happen that at some places where the given upper and lower boundary were the same, the new upper and lower boundaries will

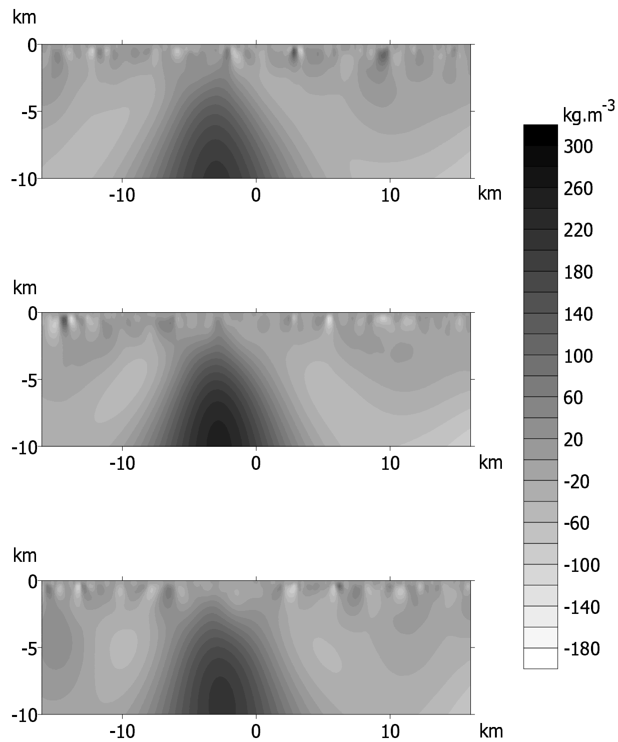

Fig. 8. Calculated χ-density for x = -1 km, x = 3 km and x = 7 km (horizontal axis y, vertical axis z).

be different (the body appears at this place); the converse can also happen (the body will disappear at this place). This process of finding successive shapes of the body (with its density kept constant) can be repeated until the match of the original χ-density and the model χ-density is satisfactory.

The calculation of the shape of the anomalous body has to begin with some starting model. This model can be chosen in many ways; one possibility is to choose the domain where the original χ-density is greater (or smaller) than some given value (depending on the sign of the χ-density within this domain), thus to copy the shape of some level of the original χ-density (of course, the body has to be defined also in the horizontal neighbourhood of this domain in order to allow its enlargement by the subsequent

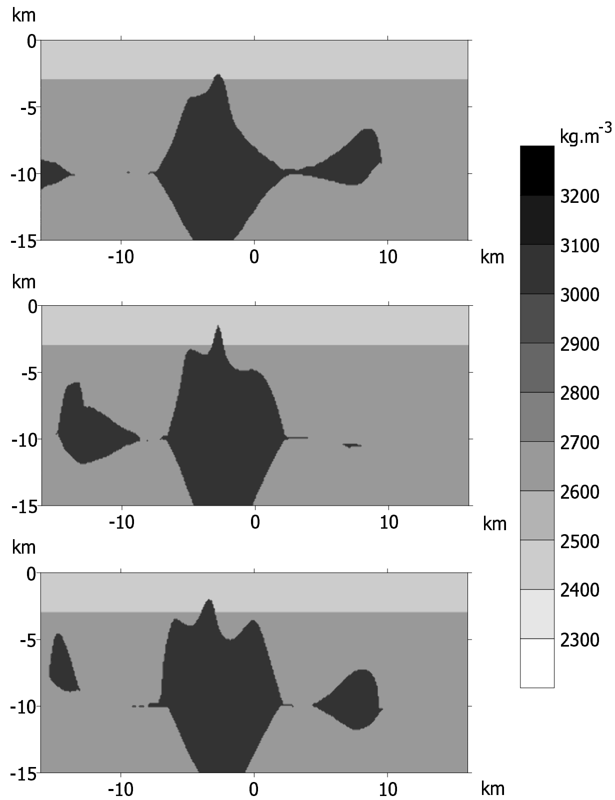

Fig. 9. Calculated density distribution for model body with density 3000 kg.m^{-3} for x = -1 km, x = 3 km and x = 7 km (horizontal axis y, vertical axis z).

calculation). Another possibility is to choose the body as the zero width horizontal prism lying at the depth of the extremum of the original χ-density; although this seems to be strange, it is the simplest possible starting model and it behaves in the calculation process very well (as was actually shown).

In the actual calculation we have used the background consisting of two horizontal layers: the upper from the surface to the depth 3 km with density 2470 kg.m^{-3}, and the lower below 3 km with density 2670 kg.m^{-3}. The density of the anomalous body was chosen in four variants to be 3000, 3100, 3200 and 3300 kg.m^{-3}, respectively. The starting model was chosen as the zero width horizontal prism with depth 10 km; this was because the χ-density has the extremum at the maximal depth of its calculation and the starting body defined by some level of χ-density would have its centre well above the depth 10 km (thus it would be shifted upwards). Another complication was that the lower boundary of the body can lie at many places well below the depth 10 km, for which the χ-density was calculated. In this case it is not possible to compare locally the χ-density of the model with the original χ-density; this was resolved by comparing the former with the original χ-density at the depth 10 km.

For every starting model 48 iterations of the process described above were performed. In every case the shape of the body has changed smoothly from the previous to the next iteration. There were several cases that parts of the body were moved upwards from the original depth 10 km, which shows that the iterative process works properly. For the body from the last iteration the corresponding χ-density was calculated in the same volume where the original χ-density was known; then it was possible to subtract these two χ-densities and thus find the resulting χ-density after removing the anomalous body from the data. This resulting χ-density was very small in the domain of the body interior; for example, in the case of the body with density 3000 kg.m^{-3}, the interior of the body contained 191412 points (of calculation of the χ-density) and the rms value of the resulting χ-density within the body was only 2.7 kg.m^{-3}. The shape of this body is presented in Figs. 3, 5, 7, 9.

The calculation of the shape of the anomalous body was a time-consuming process: one iteration took (depending on the number of points where the body has nonzero vertical dimension) up to 113 min; thus the whole iterative calculation took more than 3 days.

4. Discussion of the results

The anomalous bodies in all four variants seem to be realistic: there are (with one exception mentioned below) no strange features. As it was expected, the volume of the body decreases with increasing density. Although the χ-density was calculated only to the depth 10 km, it was possible to determine the lower boundary of the main anomalous body; this body reaches (depending on the density) to the depths of 15 – 25 km, what can be considered as satisfactory, as the body can be in fact a local elevation of the otherwise horizontal layer with upper boundary in approximately such depth. The upper boundaries of bodies show several peaks reaching well into the upper layer (depth less than 3 km). For the moment it can be hardly suggested whether this fact corresponds to reality or not. In all four variants there is one sharp narrow peak in the western part of the anomalous body reaching (dependent on density) almost to the surface. This feature is very suspect, but it can be explained as follows.

For an isolated inhomogeneity with depth (of its mass centre) d and mass m, the corresponding χ-density has the main extremum approximately at the depth d and its value is proportional to m/d^3. If the (difference) density of the inhomogeneity is ρ and its mean radius is a, the value of the main extremum of χ-density is proportional to $\rho a^3/d^3$. This has the consequence that for fixed density the main extremum is of the same order if we increase the mean radius and the depth of the inhomogeneity in the same proportion. Therefore, relatively small inhomogeneity near the surface can produce the χ-density with the main extremum comparable to much bigger inhomogeneity located much deeper. As it was mentioned above, the χ-density of an isolated body has a long tail oriented downwards. Then it may happen that some small body located near the surface above the main anomalous body can produce such χ-density that the two extremal domains corresponding to these two bodies are connected by the tail of the smaller body. This leads inevitably in the iterative calculation to the increasing of the upper boundary of the main body towards the small subsurface body.

This drawback can be removed by changing the mode of calculation of the main anomalous body. Instead of calculating first the shape of this body, it is reasonable to calculate first the shapes of smaller bodies lying above the main body (if such bodies are present; this can be inferred by inspecting the χ-density for any small subsurface features). As the χ-density depends linearly on the input, the calculated χ-density for the smaller subsurface bodies can be subtracted from the original χ-density and in this way the contribution of these bodies is removed; afterwards the shape of the main body can be calculated. This procedure was actually performed, but the result is not quite satisfactory, as the sharp peak still exists, although it lies deeper. Therefore it seems that its presence has another ground.

With this respect it has to be noted that the calculated χ-density shows, near the surface (in depths less than 1 km), a number of very small local inhomogeneities distributed randomly over the whole area. The origin of these features is unknown; it was suspected that they are present because the effect of mass between the real earth surface and the height of the origin was not accounted for. However, a variant was calculated where this effect was removed, but the most of the small inhomogeneities remained there. Therefore it is possible that these features are produced by errors in the original data: as by the calculation of χ-density according to formula (1) the input function is derivated twice, a small error at a single measurement point can produce a spurious feature in the χ-density which looks like a small subsurface inhomogeneity.

Finally, we can resume the main sources of uncertainty and errors of the calculated models of the anomalous body. The first one is the fact that the inverse method is suitable only for a plane surface and the effects of the topography are not easily removable. The second one is that the area of the original measurement data was too small to calculate safely to the depth of 10 km (not to speak about deeper region). And the third one, the data were not homogeneous enough and may contain errors.

5. Conclusion

From the theoretical point of view, this first test of the inverse method proposed by the author in his work *Pohánka (1998)* has brought two consequences: first, that the method works well numerically, and second, that the method has to be changed substantially to include the procedure of finding the shapes of anomalous bodies. The procedure of extracting information from the surface gravitational effect has thus two steps: in the first step the χ-density is calculated, while in the second step the shapes of the anomalous bodies are calculated succesively. The exact order of removing the contributions of these bodies has to be found experimentally; of course, the most prominent and the subsurface inhomogeneities have the priority. There are open possibilities to create more sophisticated methods for finding the shape of anomalous body (or even of more bodies simultaneously).

In this respect the neglecting of the first term on the rhs of formula (24) of *Pohánka (1998)* (see Section 2) has to be mentioned. This term was introduced for accounting the information about the surface value of density. The results of the previous sections show that in the case of anomalous bodies located well below the surface this term is not necessary. The situation may be different for bodies whose upper boundary touches the surface, but it is possible that also in this case the method as described in this article can be applied and the information about the surface density can be used directly by the definition of the shape of such bodies (by fixing the upper boundary in the areas where the body has to touch the surface to the zero depth).

Finally, it should be suitable to give the new method a name: as in both steps there are used in a substantial way certain harmonic functions (and their properties), we propose to call the new two step method shortly as the harmonic inversion method.

Acknowledgments. The author is grateful to VEGA, the Slovak Grant agency (grant No. 2/7059/01) for the partial support of this work.

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