Multiple-linear regression to best-estimate of gravity parameters related to simple geometrical shaped structures

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Abstract: A new interpretative approach is proposed to best-estimate of gravity parameters related to simple geometrical shaped structures such as a semi-infinite vertical cylinder, an infinite horizontal cylinder, and a sphere like structures. The proposed technique is based on the multiple-linear regression oriented towards estimating the model parameters, e.g., the depth from the surface to the center of the buried structure (sphere or infinite horizontal cylinder) or the depth from the surface to the top of the buried object (semi-infinite vertical cylinder), the amplitude coefficient, and the horizontal location from residual gravity anomaly profile. The validity of the proposed approach is firstly demonstrated through testing different synthetic data set corrupted and contaminated by a white Gaussian random noise level. The theoretical synthetic obtained results obviously show that the estimated parameters values, derived by the proposed technique are close to the assumed true parameters values. This approach is applied on five real field residual gravity anomalies taken from Cuba, Sweden, Iran, USA, and Germany, where the efficacy of this new approach is consequently proven. A comparable and acceptable agreement is noticed between the results derived by this proposed approach and those obtained from the real field data information.

Key words: Gravity anomaly, sphere-like structure, semi-infinite vertical cylinder-like structure, infinite horizontal cylinder-like structure, multiple-linear regression

1. Introduction

The geophysical gravity method is largely used in gas and oil exploration. Most of the geological structures in oil and mineral exploration can be approximated by simple geological structures such as a sphere, a semi-infinite vertical cylinder, an infinite horizontal cylinder, a fault, a sheet, and a dike. Several interpretative techniques have been already developed

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to interpret gravity field anomalies, due to simple geometric models. The main objective of those techniques utilize the mentioned approximations to best-estimate the gravity parameters values, e.g., the depth to the buried body and the amplitude coefficient. The developed methods include, linear optimization-simplex algorithm (*Asfahani and Tlas, 2015*), neural network modeling (*Abedi et al., 2010*), differential evolution algorithm (*Ekinci et al., 2016*), graphical methods (*Nettleton, 1962; 1976*), ratio methods (*Bowin et al., 1986; Abdelrahman et al., 1989*), Fourier transform (*Odegard and Berg, 1965; Sharma and Geldart, 1968*), Euler deconvolution (*Thompson, 1982*), neural network (*Elawadi et al., 2001*), Mellin transform (*Mohan et al., 1986*), least-squares minimization approaches (*Gupta, 1983; Lines and Treitel, 1984; Abdelrahman, 1990; Abdelrahman et al., 1991; Abdelrahman and El-Araby, 1993; Abdelrahman and Sharafeldin, 1995a*), Werner deconvolution (*Hartmann et al., 1971; Jain, 1976*).

Werner deconvolution technique is extended by *Kilty (1983)* to analyze the gravity data using both the residual anomaly and its first and second horizontal derivatives. *Ku and Sharp (1983)* further refined the method by using iteration for reducing and eliminating the interference field and then applied Marquardt's non-linear least squares method to further refine automatically the first approximation provided by deconvolution. A new automatic technique, called AN-EUL, and based on a combination of the analytic signal and the Euler deconvolution method is introduced by *Salem and Ravat (2003)* to interpret the magnetic data. Both the location and the approximate geometry of a magnetic source can be deduced by the use of the AN-EUL. *Fedi (2007)* proposed a method called depth from extreme points (DEXP) to interpret any potential field through describing the theory for the gravity and magnetic fields and their derivatives for any order. The DEXP technique allows the estimation of the source depths, density and structural index from the extreme points of a 3D field scaled according to specific power laws of the altitude. An alternative method is presented by *Salem and Smith (2005)* to estimate both the depth and model type using the first order local wave number approach, without the need for third order derivatives of the field. A normalization of the first order local wave-number anomalies is achieved in their method, where a generalized equation is constructed to estimate the depth of some 2D magnetic sources regardless of the source structure.

Silva and Barbosa (2003) derived the analytical estimators for the horizontal and vertical source position in 3D Euler deconvolution as a function of the x, y, and z derivatives of the magnetic anomaly within a data window. *Barbosa et al. (1999)* proposed a new criterion, based on the correlation between the total magnetic field anomaly and the estimates of an unknown base level to determine the structural index, *Salem et al. (2008)* developed a new method based on derivatives of the tilt angle to interpret the gridded magnetic data, where a simple linear equation, similar to the 3D Euler equation is obtained. Their approach evaluates both the horizontal location and the depth of magnetic bodies, without specifying prior information about the nature of the sources. *Fedi et al. (2009)* proposed a new method based on a 3D multiridge analysis of potential field. Their method assumes a 3D subset in the harmonic region and analyzes the behavior of the potential field ridges, which are built by joining extreme points of the field computed at different altitudes.

Few techniques only however, have treated the determination of shape of the buried structure. These techniques include, for example, Walsh transform (*Shaw and Agarwal, 1990*), least-squares methods (*Abdelrahman and Sharafeldin, 1995b; Abdelrahman et al., 2001a,b*), constrained and penalized nonlinear optimization technique (*Tlas et al., 2005*). The determination of the depth, shape factor, and amplitude coefficient of the buried structure is generally achieved by those methods from residual gravity anomaly, where the accuracy of the results, gathered by them, depend on the accuracy in which the residual anomaly can be separated and isolated from the observed gravity anomaly.

A simple and easy interpretative approach based on the multiple-linear regression is proposed to interpret residual gravity field anomalies and to best-estimate the model parameters values, e.g., the depth to the top or to the center of the body, the horizontal location and the amplitude coefficient related to a buried sphere or a semi-infinite vertical cylinder or an infinite horizontal cylinder-like structure.

The validity of the new proposed approach is demonstrated using synthetic data set corrupted and contaminated by white Gaussian random noise levels of 15% and 25%. The theoretical obtained results clearly show that the estimated parameter values derived by this approach are very close to the assumed true values of parameters.

The accuracy of the new proposed approach is also proven through interpreting five real field gravity anomalies, taken from Cuba, Sweden, Iran, USA, and Germany. The obtained results indicate acceptable and comparable agreements between the results derived by the proposed technique and those obtained by other interpretation methods. The depth obtained by the proposed method is moreover found to be in high accordance with that obtained from the real field data information.

2. Theory

A theoretical and synthetic residual gravity anomaly related to various geological models such as a sphere, a semi-infinite vertical cylinder and an infinite horizontal cylinder are treated to demonstrate the validity and the applicability of the new proposed approach.

2.1. Interpretation of residual gravity anomaly due to simple geometrical models

The general expression of the residual gravity anomaly (V) at any point $M(x)$ along the x-axis of a semi-infinite vertical cylinder-like structure, an infinite horizontal cylinder-like structure and a sphere-like structure, in a Cartesian coordinate system (Fig. 1) can be written according to *Gupta (1983)*, and *Asfahani and Tlas (2015)* as:

$$V(x_i) = \frac{k}{\left((x_i - x_0)^2 + z^2\right)^q}\,, \quad i = 1, ..., N\,, \tag{1}$$

where q is the geometrical shape factor of the buried structure given in discrete values as follows: $q = 1.5$ for a sphere, $q = 0.5$ for a semi-infinite vertical cylinder and $q = 1$ for an infinite horizontal cylinder, x_0 is the horizontal location of the buried body, z is the depth from the surface to the center of the buried structure (sphere or infinite horizontal cylinder) or the depth from the surface to the top of the buried object (semi-infinite vertical cylinder), k is the amplitude coefficient given by the following mathematical expression: $k = \frac{4}{3}\pi G \rho r^3 z$ for a sphere, $k = \pi G \rho r^2$ for a semi-infinite vertical cylinder and $k = 2\pi G \rho r^2 z$ for an infinite horizontal cylinder, ρ is the density contrast, G is the universal gravitational constant, r is the radius, and finally $x_i,\ i = 1, ..., N$ is the horizontal position coordinate.
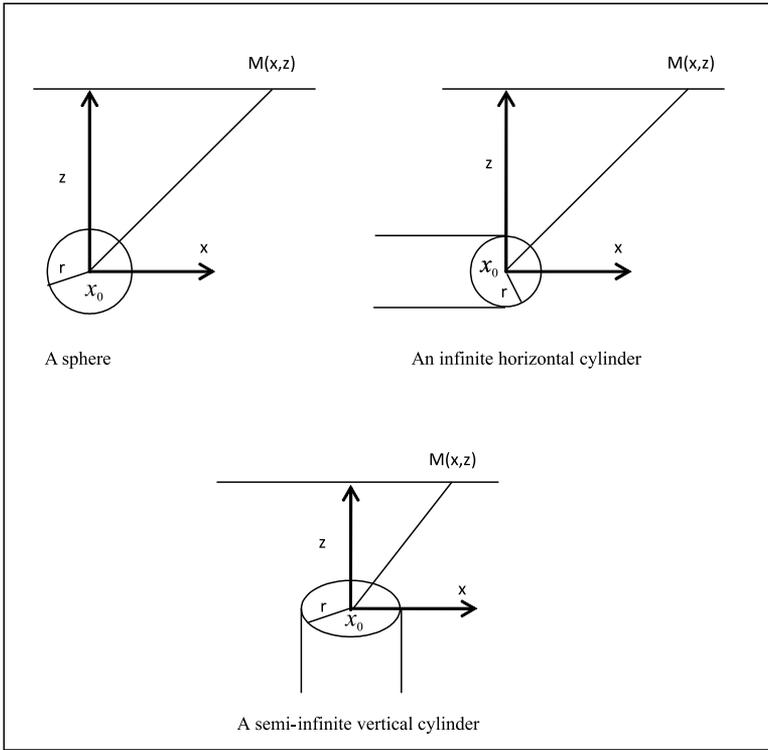
Fig. 1. Diagrams of simple geometrical structures (sphere, infinite horizontal cylinder, and semi-infinite vertical cylinder).

The set of Eq. (1) consists of N nonlinear equations in function of the three independent parameters k, x_0 and z. The term V_i will be used, for simplification, instead of the term $V(x_i)$, $i = 1, ..., N$, in the rest of the paper.

From Eq. (1), it can be easily observed that the sign of the parameter k is similar and coincident to the sign of V_i, $i = 1, ..., N$.

Taking the absolute values of both sides of Eq. (1) we find:

$$|V_i| = \frac{|k|}{\left((x_i - x_0)^2 + z^2\right)^q}, \quad i = 1, ..., N. \tag{2}$$

After the performing of some mathematical manipulation of the Eq. (2) we get:

$$|V_i|^{\frac{1}{q}} x_i^2 = 2x_0 |V_i|^{\frac{1}{q}} x_i - \left(x_0^2 + z^2\right) |V_i|^{\frac{1}{q}} + |k|^{\frac{1}{q}}, \quad i = 1, ..., N. \tag{3}$$

The introduction of the following new variables:

$$A = 2x_0, \tag{4}$$

$$B = -\left(x_0^2 + z^2\right), \tag{5}$$

$$C = |k|^{\frac{1}{q}}, \tag{6}$$

$$
\begin{aligned}
O_i &= |V_i|^{\frac{1}{q}} x_i^2, & i = 1, ..., N, \\
P_i &= |V_i|^{\frac{1}{q}} x_i, & i = 1, ..., N, \\
Q_i &= |V_i|^{\frac{1}{q}}, & i = 1, ..., N,
\end{aligned}
\tag{7}
$$

into Eq. (3) allows to write the following equation:

$$O_i = AP_i + BQ_i + C, \quad i = 1, ..., N. \tag{8}$$

Eq. (8) being linear between the variables O_i, P_i, Q_i, $i = 1, ..., N$, the unique optimal values of the coefficients A, B, and C for a specific discrete value of $q \in \{0.5, 1, 1.5\}$, a priori assumed, can be easily determined. Such determination is carried out through performing the multiple-linear regression between the dependent variable O_i, $i = 1, ..., N$ and the independent variables P_i, Q_i, $i = 1, ..., N$, with the use of one of the familiar statistical programs as Microsoft Excel or Statistica, and or through solving the following set of simultaneously linear equations by the well-known direct method of Gauss:

$$
\begin{aligned}
\left(\sum_{i=1}^{N} P_i^2\right) A + \left(\sum_{i=1}^{N} P_i Q_i\right) B + \left(\sum_{i=1}^{N} P_i\right) C &= \sum_{i=1}^{N} O_i P_i, \\
\left(\sum_{i=1}^{N} P_i Q_i\right) A + \left(\sum_{i=1}^{N} Q_i^2\right) B + \left(\sum_{i=1}^{N} Q_i\right) C &= \sum_{i=1}^{N} O_i Q_i, \\
\left(\sum_{i=1}^{N} P_i\right) A + \left(\sum_{i=1}^{N} Q_i\right) B + (N) C &= \sum_{i=1}^{N} O_i.
\end{aligned}
\tag{9}
$$

The best-estimate of the horizontal location (x_0) of the buried body can be obtained from Eq. (4), after knowing the unique optimal values of A, B and C for $q \in \{0.5, 1, 1.5\}$ as the following:

$$x_0 = \frac{A}{2} \,. \tag{10}$$

The best-estimate of the depth (z) from the surface to the center of the buried structure (sphere or infinite horizontal cylinder) or the depth from the surface to the top of the buried object (semi-infinite vertical cylinder) can be derived from Eq. (5) as:

$$z = \frac{\sqrt{|4B + A^2|}}{2} \,. \tag{11}$$

The best-estimate of the amplitude coefficient (k) can be also obtained from Eq. (6) as:

$$k = C^q \quad \text{when} \quad V_i \geq 0 \,, \quad i = 1, ..., N, \tag{12}$$

$$k = -C^q \quad \text{when} \quad V_i \leq 0 \,, \quad i = 1, ..., N. \tag{13}$$

It is useful to mention that there is no loss of generality in assuming the source geometry of the gravity anomaly as a priori known. In addition, there are no imposed restrictions on the generality of the proposed approach.

Before explaining how we can solve this ambiguity and this inconvenience, we will define the statistical criterion of preference called the Root Mean Square Error (*RMSE; Collins, 2003*). *RMSE* is based on the minimal value, between the field gravity data anomaly and the computed gravity one, obtained through using the estimated values of z, x_0 and k resulted from Eqs. (10–13) for a specific discrete value of the geometric shape factor $q \in \{0.5, 1, 1.5\}$. The mathematical formula of this statistical criterion *RMSE* is given as:

$$RMSE = \sqrt{\frac{\sum\limits_{i=1}^{N} \left(V_i \left(Observed \right) - V_i \left(Computed \right) \right)^2}{N}} \,, \tag{14}$$

where $V_i \left(Observed \right)$ and $V_i \left(Computed \right)$, $i = 1, ..., N$ are the observed and the computed gravity values at the point x_i, $i = 1, ..., N$, respectively.

In the case where the source geometry of the gravity field anomaly is unknown, the following next procedure composed of three steps should be followed:

- First, the gravity field anomaly is interpreted by adapting and assuming the source geometry as a semi-infinite vertical cylinder ($q = 0.5$), where Root Mean Square Error $RMSE\_V$ is computed using Eq. (14) with the estimated values of z, x_0 and k derived from Eqs. (10–13).

- Second, the gravity field anomaly is re-interpreted by adapting the source geometry as an infinite horizontal cylinder ($q = 1$), where the Square Correlation Coefficient $RMSE\_H$ is also computed using Eq. (14) with the estimated values of z, x_0 and k derived from Eqs. (10–13).

- Third, the gravity field anomaly is re-interpreted by assuming the source geometry as a sphere ($q = 1.5$), where the Square Correlation Coefficient $RMSE\_S$ is also computed using Eq. (14) with the estimated values of z, x_0 and k derived from Eqs. (10–13).

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and $RMSE\_S$ is selected as a convincible solution, which exactly indicates to the suitable source geometry related to the responsible gravity field anomaly.

Another statistical criterion of preference (square of correlation coefficient) can be also used to select the best optimum gravity solution. It is defined through the following mathematical expression:

$$
R^2 = \frac{\left(\sum\limits_{i=1}^{N} \left(V_i\left(Obs.\right) - \bar{V}\left(Obs.\right) \right) \times \left(V_i\left(Comp.\right) - \bar{V}\left(Comp.\right) \right) \right)^2}{\sum\limits_{i=1}^{N} \left(V_i\left(Obs.\right) - \bar{V}\left(Obs.\right) \right)^2 \times \sum\limits_{i=1}^{N} \left(V_i\left(Comp.\right) - \bar{V}\left(Comp.\right) \right)^2}, \tag{15}
$$

where $\bar{V}\left(Obs.\right)$ and $\bar{V}\left(Comp.\right)$ are the arithmetic means of $V_i\left(Obs.\right)$ and $V_i\left(Comp.\right)$, $i = 1, ..., N$ respectively. R-squared is computed for the three assumed types of geometric shapes of the buried structure, R^2_V, R^2_H and R^2_S for a semi-infinite vertical cylinder ($q = 0.5$), an infinite horizontal cylinder ($q = 1$), and a sphere ($q = 1.5$) respectively, by using Eq. (15) with the estimated values of z, x_0 and k resulted from Eqs. (10–13).

The highest one of the three reached values of R^2_V, R^2_H and R^2_S is selected as a convincible solution, which directly indicates to the suitable source geometry related to the responsible gravity field anomaly.

2.2. Interpretation of a synthetic gravity anomaly due to a sphere model with Gaussian random noise

A synthetic gravity anomaly $V(x_i)$, $i = 1, ..., N$ due to a sphere-like structure is generated from Eq. (1), by using the following values of model parameters: geometric shape factor $q = 1.5$, depth from the surface to the center of the buried spherical structure $z = 35$ m, amplitude coefficient $k = 1500$ mGal m^3, and the horizontal location $x_0 = 5$ m.

The generated synthetic anomaly is contaminated by white Gaussian random noise of 15% and 25% maximum level using continuous normal distribution, such as new additional gravity anomalies are generated (Fig. 2 and Fig. 3). This regenerated gravity anomalies are consequently interpreted by the new proposed approach. The obtained parameters (z, k, x_0) and the preference criterions $(RMSE, R^2)$ for the three structures a priori assumed; a semi-infinite vertical cylinder, an Infinite horizontal cylinder, and a sphere are summarized in detail in Table 1.

From Table 1, the lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and $RMSE\_S$ or the highest one of the three reached values of R^2_V, R^2_H and R^2_S clearly indicates that the suitable source geometry related to the responsible contaminated synthetic gravity anomaly is a sphere.

The results presented in Table 1 show that the estimated parameter values, obtained by the proposed method, are very close to the true assumed
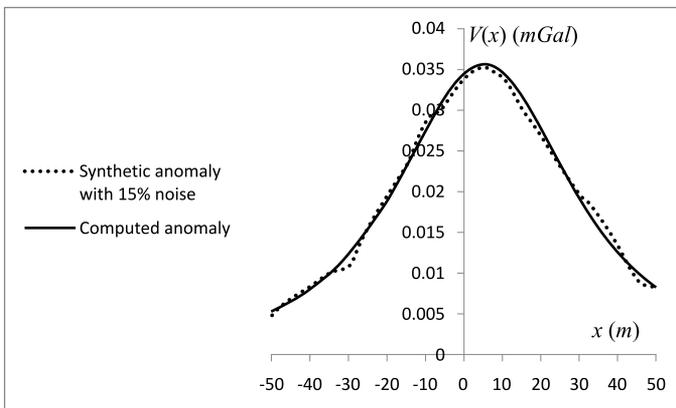

Fig. 2. Diagrams of the computed anomaly and synthetic data set due to a sphere with adding a maximum of 15% random noise.
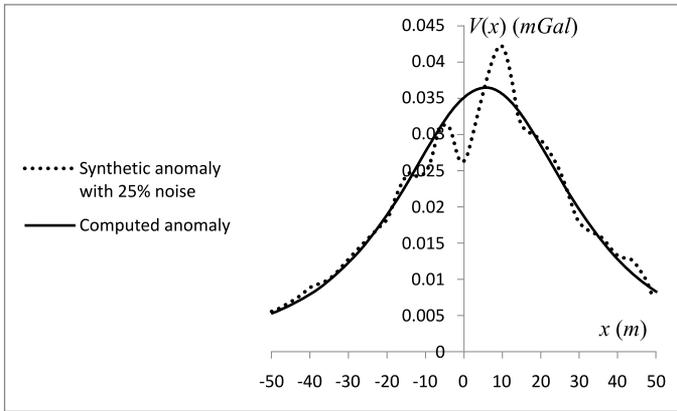
Fig. 3. Diagrams of the computed anomaly and synthetic data set due to a sphere with adding a maximum of 25% random noise.

Table 1. Interpretation of a synthetic gravity anomaly with 15% and 25% maximum level of Gaussian random noise.

Source geometric shape	Model parameters	True values of model parameters	Estimated values of model parameters with maximum 15% random noise	Estimated values of model parameters with maximum 25% random noise
Semi-infinite vertical cylinder ($q = 0.5$)	z (m)	35	9.6802	9.2577
	k (mGal m)	1500	0.4400	0.4402
	x_0 (m)	5	5.7053	6.0398
	$RMSE$ (mGal)	–	0.0040	0.0045
	R^2	–	0.9005	0.8659
Infinite horizontal cylinder ($q = 1$)	z (m)	35	24.1526	23.6664
	k (mGal m^2)	1500	21.9485	21.6798
	x_0 (m)	5	5.5931	5.8338
	$RMSE$ (mGal)	–	0.0025	0.0028
	R^2	–	0.9710	0.9314
Sphere ($q = 1.5$)	z (m)	35	34.8422	34.3145
	k (mGal m^3)	1500	1505	1472
	x_0 (m)	5	5.5.2868	5.4353
	$RMSE$ (mGal)	–	0.0019	0.0027
	R^2	–	0.9753	0.9342

values of parameters. This clearly proves the efficiency of the new proposed approach.

3. Application to field data

The new proposed approach is applied to interpret five field residual gravity anomalies measured over various geological structures. The five field gravity anomalies are interpreted according to the three different geological structures, e.g., a sphere, an infinite horizontal cylinder, and a semi-infinite vertical cylinder. The resulting model with the lowest reached value of $RMSE$ and the highest reached value of R^2 is selected as the best and the suitable model solution for estimating the parameters of the field residual gravity anomaly.

3.1. Interpretation of the Chromites filed residual gravity anomaly

Fig. 4 shows a normalized residual field gravity anomaly measured over a chromites deposit in Camaguey province, Cuba (*Robinson and Coruh, 1988*). The gravity anomaly is interpreted by the new proposed approach by assuming a priori the source geometry as a semi-infinite vertical cylinder,
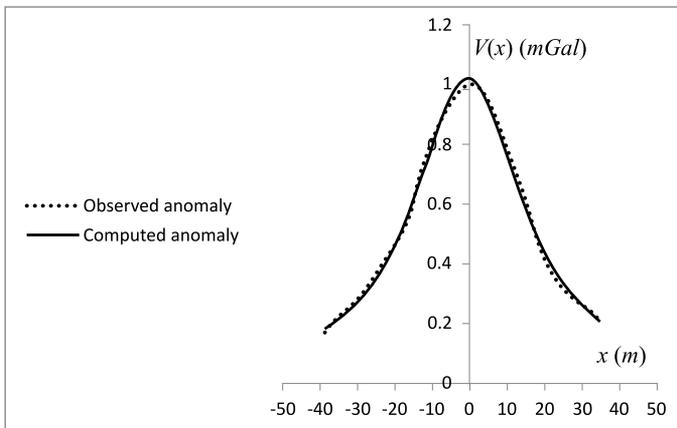

Fig. 4. Normalized residual gravity field anomaly over a chromites deposit, Camaguey province, Cuba. The evaluated curve by the proposed approach is presented for an infinite horizontal cylinder model.

an infinite horizontal cylinder, and a sphere. All acquired results are shown
in Table 2.

Table 2. Interpretation of the Chromites field residual gravity anomaly, Cuba.

Model parameters	Semi-infinite vertical cylinder	Infinite horizontal cylinder	Sphere
z (m)	7.2116	17.7302	25.5077
k	8.6920 (mGal m)	321.3769 (mGal m^2)	16 121 (mGal m^3)
x_0 (m)	–0.6609	–0.5827	–0.6541
$RMSE$ (mGal)	0.0742	0.0153	0.0219
R^2	0.9443	0.9971	0.9944

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and
$RMSE\_S$ with the highest one of the three reached values of R^2_V, R^2_H
and R^2_S are obtained for the infinite horizontal cylinder, meaning that, the
field residual gravity anomaly could be preferably modeled as an infinite hor-
izontal cylinder. The depth obtained in this case ($z = 17.7302$ m) is found
to be in a good agreement with that obtained from drill-hole information
($z = 21$ m) (*Robinson and Coruh, 1988*). The computed gravity anomaly
is drawn according to the estimated values of infinite horizontal cylinder
model parameters as shown in Fig. 4. The comparison between field and
computed anomalies shows a close agreement between them, which certainly
attests the validity of the new proposed approach.

3.2. Interpretation of the Karrbo field residual gravity anomaly

Fig. 5 shows a field residual gravity anomaly of a profile 25.6 m length
obtained over the two-dimensional pyrrhotite ore, Karrbo, Vastmanland,
Sweden (*Shaw and Agarwal, 1990*). The field gravity anomaly is also inter-
preted by the proposed approach for the three different structures a priori
assumed, where the gathered results are summarized in Table 3.

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and
$RMSE\_S$ with the highest one of the three reached values of R^2_V, R^2_H
and R^2_S are obtained for the infinite horizontal cylinder, meaning that,
the field residual gravity anomaly could be preferably modeled as an infi-
nite horizontal cylinder. The depth obtained in this case ($z = 4.7062$ m) is
found to be in good agreement with the depth reported by *Tlas et al. (2005)*
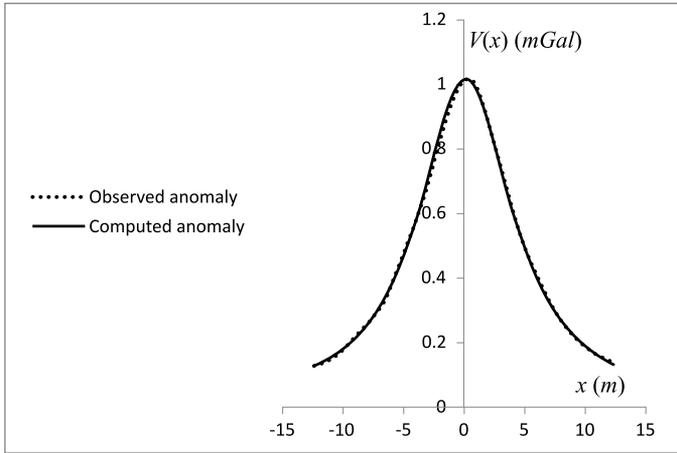
Fig. 5. Residual gravity field anomaly over the two-dimensional pyrrhotite ore, Karrbo, Vastmanland, Sweden. The evaluated curve by the proposed method is presented for an infinite horizontal cylinder model.

Table 3. Interpretation of the Karrbo field residual gravity anomaly, Sweden.

Model parameters	Semi-infinite vertical cylinder	Infinite horizontal cylinder	Sphere
z (m)	1.5839	4.7062	7.057
k	2.1303 $(\mathrm{mGal\,m})$	22.5104 $(\mathrm{mGal\,m^2})$	331.0846 $(\mathrm{mGal\,m^3})$
x_0 (m)	0.1693	0.1772	0.1767
$RMSE$ (mGal)	0.0947	0.0055	0.0279
R^2	0.9252	0.9996	0.9922

($z = 4.82$ m), *Asfahani and Tlas (2015)* ($z = 4.7$ m), *Shaw and Agarwal (1990)* ($z = 5.8$ m), and *El-Araby (2000)* ($z = 5.23$ m). The computed gravity anomaly is drawn according to the estimated values of infinite horizontal cylinder model parameters as shown in Fig. 5. The comparison between field and computed anomalies shows a close agreement between them, which attests the capability and the validity of the new suggested approach.

3.3. Interpretation of the Dehloran field residual gravity anomaly

Fig. 6 shows a field residual gravity anomaly obtained over an area located in West of Iran in the Zagros tectonic zone, Iran (*Abedi et al., 2010*). The
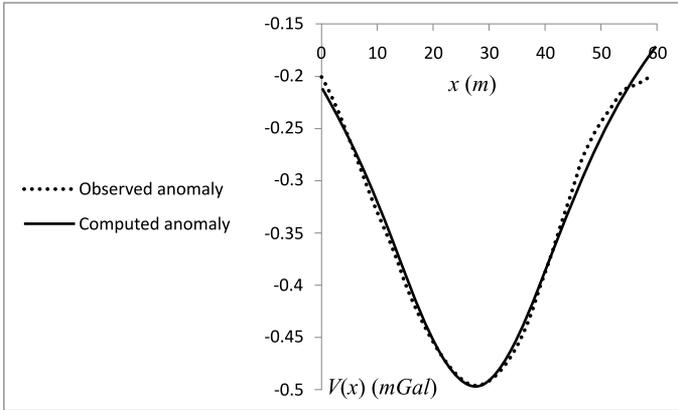
Fig. 6. Residual gravity field anomaly over an area located in West of Iran in the Zagros tectonic zone, Iran. The evaluated curve by the proposed method is presented for an infinite horizontal cylinder model.

field gravity anomaly is interpreted by the proposed approach, where the interpretative results are presented in Table 4.

Table 4. Interpretation of the Dehloran field residual gravity anomaly, Iran.

Model parameters	Semi-infinite vertical cylinder	Infinite horizontal cylinder	Sphere
z (m)	12.7586	23.6345	31.4088
k	-6.657 (mGal m)	-277.2779 (mGal m^2)	$-15\,142$ (mGal m^3)
x_0 (m)	27.1784	27.4276	27.5829
$RMSE$ (mGal)	0.0147	0.0097	0.0122
R^2	0.9814	0.9915	0.9865

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and $RMSE\_S$ with the highest one of the three reached values of R^2_V, R^2_H and R^2_S are obtained for the infinite horizontal cylinder, meaning that, the field residual gravity anomaly could be preferably modeled as an infinite horizontal cylinder. The depth obtained in this case ($z = 23.6345$ m) is found to be in good agreement with that reported by (*Abedi et al., 2010*), by using three different interpretation methods, the normalized method ($z = 23.73$ m), the least-squares method ($z = 23.3145$ m), the neural network method ($z = 22.8$ m), and also with that obtained from drill-hole information ($z = 23$ m) (*Abedi et al., 2010*).

The computed gravity anomaly is drawn according to the estimated values of infinite horizontal cylinder model parameters as shown in Fig. 6. The comparison between field and computed anomalies shows a close agreement between them, which attests the validity of the suggested approach.

3.4. Interpretation of the Humble field residual gravity anomaly

Fig. 7 shows a field residual gravity anomaly obtained over an area located near the town of Humble in north-eastern Harris County on the Upper Gulf Coast of Texas, USA (*Nettleton, 1962; Abdelrahman and Gobashy, 2017*). The field gravity anomaly is newly reinterpreted by the proposed approach. The results related to this residual gravity anomaly are shown in Table 5.
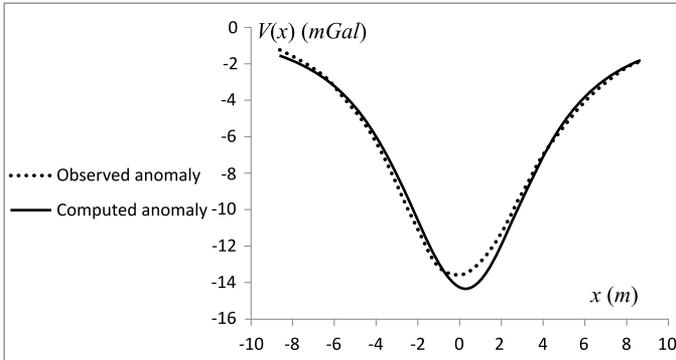

Fig. 7. Residual gravity field anomaly over an area located near the town of Humble in north-eastern Harris County on the Upper Gulf Coast of Texas, USA. The evaluated curve by the proposed method is presented for a sphere model.

Table 5. Interpretation of the Humble field residual gravity anomaly, USA.

Model parameters	Semi-infinite vertical cylinder	Infinite horizontal cylinder	Sphere
z (km)	1.0435	3.2087	4.8192
k	-22.4903 (mGal km)	-159.5463 (mGal km^2)	-1600.2 (mGal km^3)
x_0 (km)	0.1423	0.2517	0.2941
$RMSE$ (mGal)	2.3061	0.7804	0.4319
R^2	0.8329	0.9744	0.9908

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and $RMSE\_S$ with the highest one of the three reached values of R^2_V, R^2_H

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and R^2_S has been obtained for the sphere, meaning that, the field residual gravity anomaly could be preferably modeled as a sphere. The depth obtained in this case ($z = 4.8192$ km) is found to be in a good agreement with that reported by *Nettleton (1962)* and by *Abdelrahman and Gobashy (2017)* ($z = 5.05$ km) using a statistical approach.

The computed gravity anomaly is drawn according to the estimated values of sphere model parameters as shown in Fig. 7. The comparison between field and computed anomalies shows a close agreement between them, which proves the validity of the proposed approach.

3.5. Interpretation of the Wathlingen field residual gravity anomaly

Fig. 8 shows a field residual gravity anomaly obtained over an area situated in the southern part of the North-West German Basin, Germany (*Dubey et al., 2014; Abdelrahman and Gobashy, 2017*). The field gravity anomaly is reinterpreted by the new proposed approach, where the results are shown in Table 6.

The lowest one of the three reached values of $RMSE\_V$, $RMSE\_H$ and $RMSE\_S$ with the highest one of the three reached values of R^2_V, R^2_H and R^2_S are obtained for the semi-infinite vertical cylinder, meaning that,
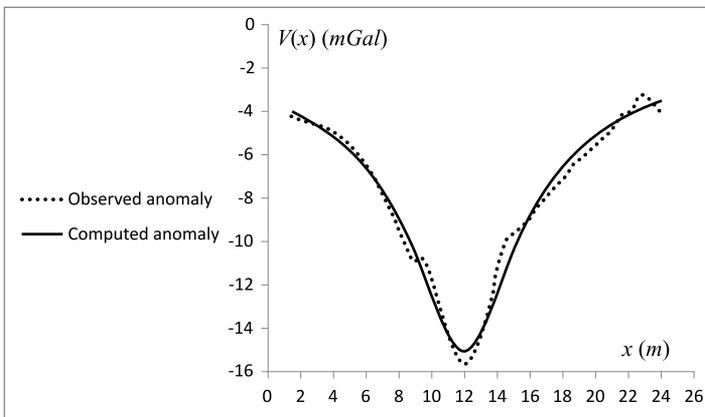

Fig. 8. Residual gravity field anomaly over an area situated in the southern part of the North-West German Basin, Germany. The evaluated curve by the proposed method is presented for a semi-infinite vertical cylinder model.

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Table 6. Interpretation of the Wathlingen field residual gravity anomaly, Germany.

Model parameters	Semi-infinite vertical cylinder	Infinite horizontal cylinder	Sphere
z (km)	2.9155	6.2657	8.7195
k	−43.8499 (mGal km)	−541.4824 (mGal km^2)	−8885.5 (mGal km^3)
x_0 (km)	11.9780	12.1696	12.2627
$RMSE$ (mGal)	0.5183	0.9208	1.0871
R^2	0.9828	0.9448	0.9228

the field residual gravity anomaly could be preferably modeled as a semi-infinite vertical cylinder. The depth obtained in this case ($z = 2.9155$ km) is found to be in a good agreement with that reported by *Dubey et al. (2014)* and by *Abdelrahman and Gobashy (2017)* by using a statistical approach ($z = 3.2$ km).

The computed gravity anomaly is drawn according to the estimated values of semi-infinite vertical cylinder model parameters as shown in Fig. 8. The comparison between field and computed anomalies shows a close agreement between them, which proves the validity of the proposed approach.

4. Pseudo-code of the proposed interpretation method

A pseudo-code of the proposed method is explained and illustrated in this section for interpreting residual gravity anomalies related to three different types of structures, e.g., a semi-infinite vertical cylinder, an infinite horizontal cylinder, and a sphere. Such pseudo-code procedure facilitates and aids to put the interpretation method into a computer code.

The statistical criteria of preference Root Mean Square Error ($RMSE$) defined in Eq. (14) and the square of correlation coefficient (R^2) defined in Eq. (15) are used in the proposed method in order to choose the suitable source geometry related to the responsible gravity field anomaly.

The pseudo-code procedure
$V_i\,(Observed)\,,\ i = 1, ..., N$ is the observed data,
q is the shape factor of the buried structure,
$RMSE$ is the root mean square error,
R^2 is the square of correlation coefficient.
begin
    for $q = 0.5,\ 1,\ 1.5$ **do**

  Find the pair values of $RMSE\_V$, and R^2_V for a semi-infinite vertical
  cylinder model ($q = 0.5$)
  Find the pair values of ($RMSE\_H, R^2\_H$) for an infinite horizontal
  cylinder ($q = 1$)
  Find the pair values of ($RMSE\_V, R^2\_V$) for a sphere ($q = 1.5$)
 end for
 Find the parameters $RMSE = \min\left\{RMSE\_V, RMSE\_H, RMSE\_S\right\}$, and
 $R^2 = \max\left\{R^2\_V, R^2\_H, R^2\_S\right\}$
 if $RMSE = RMSE\_V$ or $R^2 = R^2\_V$, **then**
  the gravity anomaly could be modeled as a semi-infinite vertical cylinder,
  end the procedure
 else
  the gravity anomaly is not preferably to be modeled as a semi-infinite
  vertical cylinder, **continue**
 end if
 if $RMSE = RMSE\_H$ or $R^2 = R^2\_H$, **then**
  the gravity anomaly could be modeled as an infinite horizontal cylinder,
  end the procedure
 else
  the gravity anomaly is not preferably to be modeled as a an infinite
  horizontal cylinder, **continue**
 end if
 if $RMSE = RMSE\_S$ or $R^2 = R^2\_S$, **then**
  the gravity anomaly could be modeled as a sphere, **end the procedure**
 else
  the gravity anomaly is not preferably to be modeled as a sphere
 end if
end begin
end the pseudo-code

5. Conclusion

The new proposed approach is applied to interpret five real field gravity
anomalies from Cuba, Sweden, Iran, USA and Germany, where satisfactory
results were obtained.

An acceptable agreement between the results derived by this approach and those obtained by other interpretative methods is shown, which manifests the validity of the proposed approach.

Moreover, the depth obtained by such the new proposed approach is found to be in an acceptable accordance with that obtained by the real field data information.

The new proposed approach could be easily put in MATLAB code or in Excel sheet. It is recommended to apply this method for routine analysis of gravity anomalies for determining the best-estimate values of parameters related to spheres, semi-infinite vertical cylinder and infinite horizontal cylinder-like structures.

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