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An analysis of methods for gravity determination and their utilization for the calculation of geopotential numbers in the Slovak national levelling network

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Abstract: The vertical reference system in the Slovak Republic is realized by the National Levelling Network (NLN). The normal heights according to Molodensky have been introduced as reference heights in the NLN in 1957. Since then, the gravity correction, which is necessary to determine the reference heights in the NLN, has been obtained by an interpolation either from the simple or complete Bouguer anomalies. We refer to this method as the "original". Currently, the method based on geopotential numbers is the preferred way to unify the European levelling networks. The core of this article is an analysis of different ways to the gravity determination and their application for the calculation of geopotential numbers at the points of the NLN. The first method is based on the calculation of gravity at levelling points from the interpolated values of the complete Bouguer anomaly using the CBA2G_SK software. The second method is based on the global geopotential model EGM2008 improved by the Residual Terrain Model (RTM) approach. The calculated gravity is used to determine the normal heights according to Molodensky along parts of the levelling lines around the EVRF2007 datum point EH-V. Pitelová (UELN- 1905325) and the levelling line of the 2^{nd} order NLN to Kráľova hoľa Mountain (the highest point measured by levelling). The results from our analysis illustrate that the method based on the interpolated value of gravity is a better method for gravity determination when we do not know the measured gravity. It was shown that this method is suitable for the determination of geopotential numbers and reference heights in the Slovak national levelling network at the points in which the gravity is not observed

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directly. We also demonstrated the necessity of using the precise RTM for the refinement of the results derived solely from the EGM2008.

Key words: Slovak National Levelling Network, complete Bouguer anomaly, GGM RTM approach

1. Introduction

The basic framework of the National Levelling Network (NLN) consists of the 1^{st} order levelling lines, which were designed as closed polygons with the length of approx. 280 km. It contains approximately 11000 points and represents 3787 km of the levelling lines (*Hudec and Ferianc, 2007*). There are levelling lines of the 2^{nd} order embedded into the 1^{st} order areas with over 24000 points, which represent 9590 km of the levelling lines (*Hudec and Ferianc, 2007*).

The Slovak Republic has participated in the UELN (United European Levelling Network) project since 1994. In that year Slovakia for the first time contributed to the data centre with the data (the geopotential differences) from the repeated levelling of the 1^{st} order points in the CSULN.

After 2007 the EVRF2007 was adopted as the new realization of the EVRS. The Slovak Republic contributed with geopotential differences of the 1^{st} order of the NLN to the realization of the EVRF2007. The levelling elevations were obtained from precise levelling measurements and the gravity values came mostly from the interpolation of the complete Bouguer anomaly from map 1:25000 (*Grand et al., 2001*). The results of the EVRF2007 adjustment for the area of the Slovak Republic represent 168 points with the geopotential numbers (*Bublavý and Droščák, 2015*).

In the context of the connection to the EVRS, it is necessary to modernize the reference height computation in the NLN. The reference normal heights are still calculated by the original method using components of the gravity correction. The complete Bouguer anomaly value used for calculation of the gravity correction is interpolated from the original gravimetric map, resulting from the gravity survey 1:25000 in the Slovak realization of the Potsdam gravimetric system S-Gr64 (*Grand et al., 2001*). The data are affected by the error of the reference level Potsdam gravity system (approx. −13.8 mGal) and by the error of the data themselves. The value of the normal gravity was calculated by the Helmert formula (1901–1909) for the Krasovsky ellipsoid. For these reasons the calculation of the normal heights in the NLN needs some actualization (using the currently valid gravimetric system S-Gr95 and the ellipsoid GRS-80 for the normal gravity field).

A new realization of the height system based on the geopotential numbers has recently been prepared. The method uses the levelling elevation in combination with the gravity at levelling points. The advantage of this approach is that both parameters can be observed in principle. But in practice the gravity at the levelling points is often not available. Therefore we want to make a comparison of three methods for the estimation of the missing gravity at the levelling points. As input we use a map of complete Bouguer anomalies, which is provided by the Earth Science Institute of the Slovak Academy of Sciences. We also employ the freely available global geopotential model (GGM) – EGM2008 (*Pavlis et al., 2012*).

The set of the testing points contains the geodetic control points within Slovakia at which the gravity was directly observed. After the gravity testing we used these values to determine the geopotential numbers and, finally, we calculated the normal heights within two testing areas: A – Pitelová and B – Kráľova hoľa.

2. Theoretical background

The physical height is in general defined as a vertical distance between reference surface and the measured point along the normal or real plumbline. In geodetic practice the method of precise levelling is most often used for the height measurements. The principle of the precise levelling is very simple (Figure 1): two vertical rods are placed on points A and B and the levelling instrument is in the middle between them. If the line $r_A - r_B$ is horizontal, the difference between the rod readings r_A and r_B is the height difference ΔH_{AB} (geometrical interpretation):

$$\Delta H_{AB} = r_A - r_B \,, \tag{1}$$

where r_A and r_B are readings on the levelling rods A and B.

For a successful realization of precise levelling we have to respect certain measurement principles defined in the existing guidelines. For instance, the

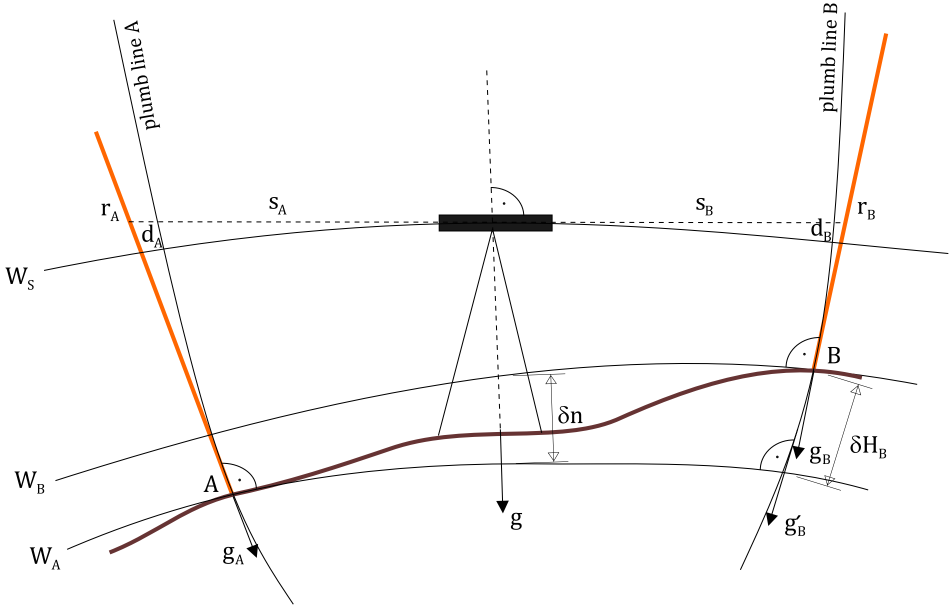

Fig. 1. Principle of levelling measurement.

levelling instrument and invar levelling rods have to be calibrated. Even if the most precise instruments and the most reliable methodology during the loop measurements are used, the sum of height differences will not be zero in general (*Hofmann-Wellenhof and Moritz, 2005*).

The difference δn is the vertical distance between the equipotential surface at the point A and the equipotential surface at the point B. This distance is not the same as the distance δH_B in the plumb line at the corresponding point B (at that point at which it is determined). This is a consequence of the fact that the equipotential surfaces converge to the poles and that they are locally corrugated. Therefore it is necessary to define the height differences through the differences of the gravity potential δW_{AB} (in accordance with Figure 1):

$$-\delta W_{AB} = g\delta n = g_B{}'\delta H_B = const.\,, \tag{2}$$

where g is gravity at the levelling station and $g_B{}'$ is gravity in the plumb line of the point B at the distance δH_B (*Hofmann-Wellenhof and Moritz, 2005*).

Thus, the potential difference can be obtained by a combination of the spirit levelling and the gravity measurement at the levelling points. Levelling without gravity measurements is then merely a geometric interpretation of the physical height definition.

The *geopotential number* has been introduced for the direct definition of the physical height. It represents the potential difference between the geoid W_0 and the equipotential surface at the measured point B:

$$W_0 - W_B = \int_0^B g \, \mathrm{d}n = C_B \,, \tag{3}$$

where C_B is a geopotential number of the point B which is independent of the levelling route.

Then, the unique physical height is the proportion of the geopotential number and some mean value of gravity (see for example *Hofmann-Wellenhof and Moritz, 2005*). For normal heights according to Molodensky we use a mean value of the normal gravity along the normal plumb line between the reference ellipsoid and telluroid i.e. $\bar{\gamma}$:

$$H_B^N = \frac{C_B}{\bar{\gamma}} \,. \tag{4}$$

2.1. Realization of the geopotential numbers

For the realization of the geopotential numbers, in accordance with (3), levelling elevations and the gravity along the plumb line between geoid and levelling point A are necessary. But the integral (3) can be approximated by summation with limits at the levelling points A and B, and the potential difference (or the geopotential difference) can be calculated:

$$W_B - W_A = \Delta W_{AB} = \Delta C_{AB} = \int_A^B g \, \mathrm{d}H \doteq \sum_A^B \bar{g} \, \Delta H_{LEV} \,, \tag{5}$$

where $\bar{g}$ is the mean value of the gravity between neighbouring points measured on the Earth's surface and is the levelled elevation increment between them. It is assumed that the geopotential number of the reference point (the first point in the levelling line) is known.

Geopotential numbers at each levelling point are computed by the summation of the partial potential differences (5) which are added to the reference value of the geopotential number.

2.2. Original method for calculation of the normal heights

Due to the missing values of the gravity at the levelling points the original method is used for practical determination of the reference heights in the NLN. This method uses the precise levelling elevations, and the effect of the gravity field is approximated trough the normal correction (*Kruis, 1957*) or later named as the gravity correction C_q (*Abelovič et al., 1990*). The gravity correction consists of two parts: *the normal orthometric correction* O_γ and *the correction term* P.

The normal orthometric correction reflects the convergence of normal equipotential surfaces towards the poles due to the Earth's flattening. It is defined by the formula (*Kruis, 1957*):

$$O_\gamma = -2\beta H_{LEV_m} \sin(2\varphi_m)\,\Delta\varphi\,, \tag{6}$$

where β is the gravity coefficient which defines the flattening of the reference ellipsoid (*Kuska, 1974*), H_{LEV_m} is the mean value (an average) of the levelling heights between the neighbouring points, $\Delta\varphi$ is the difference of their ellipsoidal latitudes and φ_m is the mean ellipsoidal latitude over the computing area.

The correction term represents the local changes of the gravity field caused by the topography and non-homogeneity of the Earth's crust in the concerned area and also transforms the normal orthometric height to the normal height (*Kruis, 1957*). For the calculation of the correction term it is necessary to use information about the gravity field along the levelling line:

$$P = \frac{1}{\gamma_m}(g-\gamma)_m \cdot \Delta H_{LEV}\,, \tag{7}$$

where γ_m is the value of the normal gravity for mean ellipsoidal latitude in the levelling section and $(g-\gamma)_m$ is the mean value of the free-air anomaly Δg_{FAA_m} between two neighbouring points. This value is obtained from the interpolated simple or complete Bouguer anomaly.

3. Practical experiment

The aim of the practical experiment was determination of the gravity values at the test points and subsequently their comparison with the directly measured gravity, which is known on each test point. Three approaches have been tested:

- computed reversely from the interpolated complete Bouguer anomaly using the CBA2G_SK software (*Marušiak et al., 2015*) – g_{CBA2G};
- computed from the EGM2008 global geopotential model (*Pavlis et al., 2012*) – $g_{EGM2008}$;
- computed from the EGM2008 with an effect of the Residual Terrain Model (RTM) – $g_{EGM+RTM}$.

The computed gravity was then used for the determination of geopotential numbers and normal heights at the points of the tested levelling lines. In this experiment the influence of the specific approaches to gravity determination on the final normal heights in the NLN was studied.

3.1. Description of the areas of study

For the purpose of the experiment a set of the geodetic control points were chosen. These points were provided by the Geodetic and Cartographic Institute Bratislava. This sample consists of the levelling points (525 items), gravimetric points (307 items), points of the National Spatial Network (228 items) and trigonometric points (43 items). Each point contains the value of the directly measured gravity in the reference Slovak Gravimetric System (S-Gr95). The positions of the points were determined mostly by interpolation from the topographic maps (levelling and gravimetric points) in the system of Unified Trigonometric Cadastral Network (S-JTSK) or by direct measurements (points of the spatial and trigonometric network) in the system ETRS89. Our own gravity measurements were added to the set on the levelling points in the areas A – Pitelová (152 points) and B – Kráľova hoľa (22 points). The positions of all these points were directly observed by GNSS RTK method with the national Slovak real-time positioning service SK*POS* (*SKPOS, 2016*). Totally there were 1277 tested points (Figure 2).

The geopotential numbers and normal heights were calculated on the points of the two levelling areas (see Figures 2 and 3):

A. Four levelling lines around the EVRF2007 datum point UELN-1905325 (EH-V. Pitelová) (Figure 3 left)
   - Banská Bystrica
   - Handlová
   - Banská Štiavnica
   - Pitelová

B. Levelling line Kráľova hoľa, which connects the point of the National Spatial Network No. 3711PP-24 with the National levelling network. It is the highest levelled points in the NLN (1942 m) (Figure 3 right).

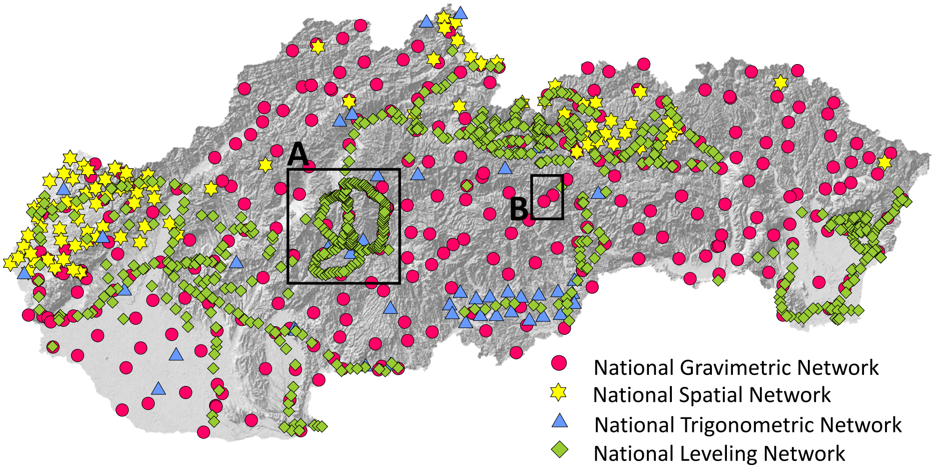

Fig. 2. Types and spatial distribution of the test points and the test levelling lines A – Pitelová and B – Kráľova hoľa.

3.2. Gravity determination

3.2.1. Gravity determination by the CBA2G_SK software

The software was developed in cooperation with G-Trend, Ltd. Company, the Earth Science Institute of the Slovak Academy of Sciences, the Department of Theoretical Geodesy of the Slovak University of Technology and

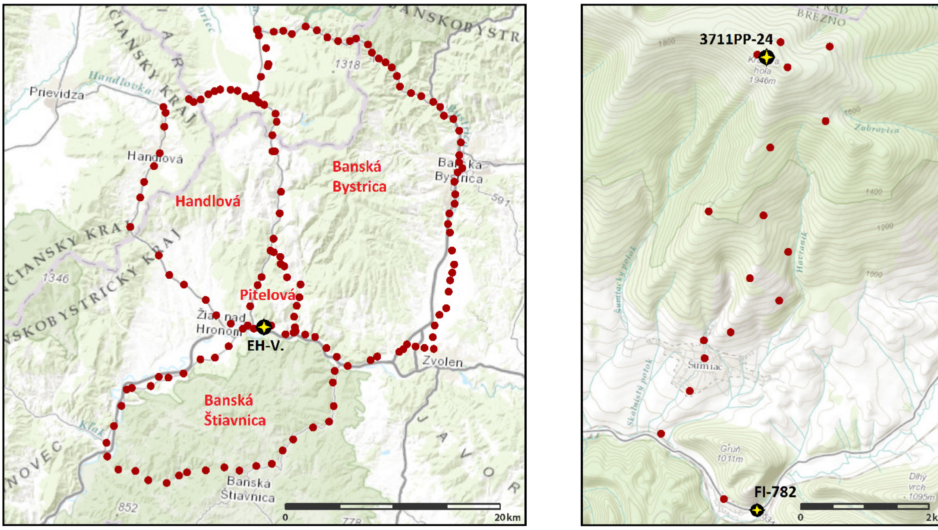

Fig. 3. More detailed overview of the tested levelling lines for the calculation of the geopotential numbers and the normal heights; left A – Pitelová and right B – Kráľova hoľa (Topographic maps provided by: http://www.arcgis.com).

Faculty of Natural Sciences of the Comenius University in Bratislava.

The CBA2G_SK software is designed for gravity recalculation from the interpolated value of the complete Bouguer anomaly. The inputs to the CBA2G_SK contain the list of coordinates (in the S-JTSK/JTSK03 or the ETRS89) and levelling heights in the system Baltic after adjustment, the grid of complete Bouguer anomaly and Digital Elevation Models (DEMs) for the necessary calculation of the terrain corrections. The output file contains the simple and complete Bouguer anomaly, the terrain corrections in the zones T1–T32, the free-air anomaly and the estimated gravity value for each of the input points. The user interface of the CBA2G_SK is shown in Figure 4.

For our experiment the new grid of complete Bouguer anomaly created on the basis of the gravity survey 1:25000 (1956–1993) was used (*Grand et al., 2001*). The data were supplemented by the detailed gravity measurements within the APVV-0194-10 project "Bouguer Anomalies of New Generation and Gravimetrical Model of Western Carpathians". The database now contains more than 319000 gravimetric points in the territory of the Slovak

Republic (*Pašteka et al., 2014*). Terrain corrections $\delta g_{top}^{0-166.7\,km}$ were calculated according to known formulas using the Toposk software (*Marušiak et al., 2013*) with the most recent DEMs in the standard calculation zones:

- zone T1: 0–250 m – DMR3 (*TOPÚ, 2012*);
- zone T2: 250–5240 m – DMR3-30 (*TOPÚ, 2012*);
- zone T31: 5240–28800 m – SRTM-3 (*Reuter et al., 2007*);
- zone T32: 28800–166735 m – SRTM-30 (*Reuter et al., 2007*).

The final gravity on the test points was calculated by CBA2G_SK according to following formula:

$$g_{CBA2G} \doteq \Delta g_{CBA} + \gamma_0 + \delta g_{faa} + \delta g_{sph}^{0-166.7\,km} - \delta g_{atm} - \delta g_{top}^{0-166.7\,km} , \qquad (8)$$

where Δg_{CBA} is an interpolated value of the complete Bouguer anomaly, γ_0 is the normal gravity on the reference ellipsoid GRS-80 calculated by the Somigliana formula (*Torge, 1989*), δg_{faa} is the free-air correction calculated using two degrees of the Taylor polynomial (*Torge, 1989*), $\delta g_{sph}^{0-166.7\,km}$ is the gravitational effect of the truncated spherical layer up to the distance

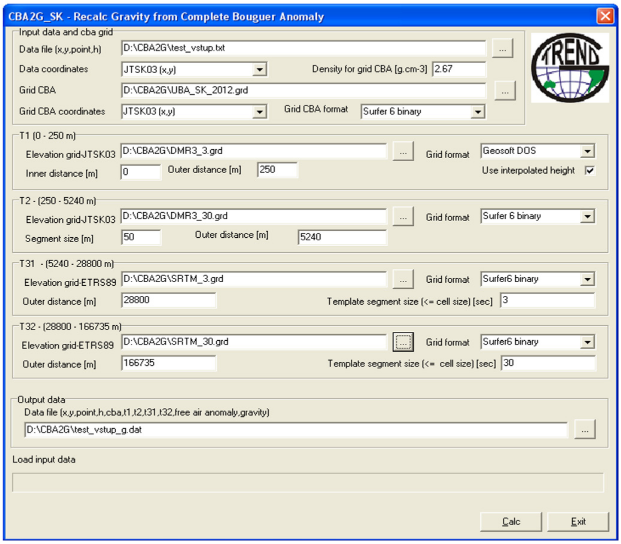

Fig. 4. User interface of the CBA2G_SK software (*Marušiak et al., 2015*).

of 166.7 km, δg_{atm} is an atmospheric correction (*Torge, 1989*) which was modified for the territory of Slovakia including the allowance for topography after *Mikuška et al. (2008)*.

The gravity g_{CBA2G} calculated by the CBA2G_SK software was compared with the directly measured gravity g_{meas} on the testing points.

The next pictures (Figures 5 and 6) show the differences in the maps with the digital elevation model in the background. The first map shows the differences, which are in the interval ±1.5 mGal. In the second map there are eight points with the differences outside this interval. The map (Figure 6) shows also types of these points. The points with the biggest differences belong to the National levelling network, where the point locations have been obtained from analogue topographical maps with accuracy of the interpolation to 45 m (*Bublavý and Droščák, 2015*). Reliability of the gravity recalculated from complete Bouguer anomaly depends greatly on the accuracy of the point position.

3.2.2. Gravity determination from EGM2008 and EGM+RTM

To model the gravity from EGM2008 ($g_{EGM2008}$), we used the zero tide version published by the EGM2008 development team. The spherical har-

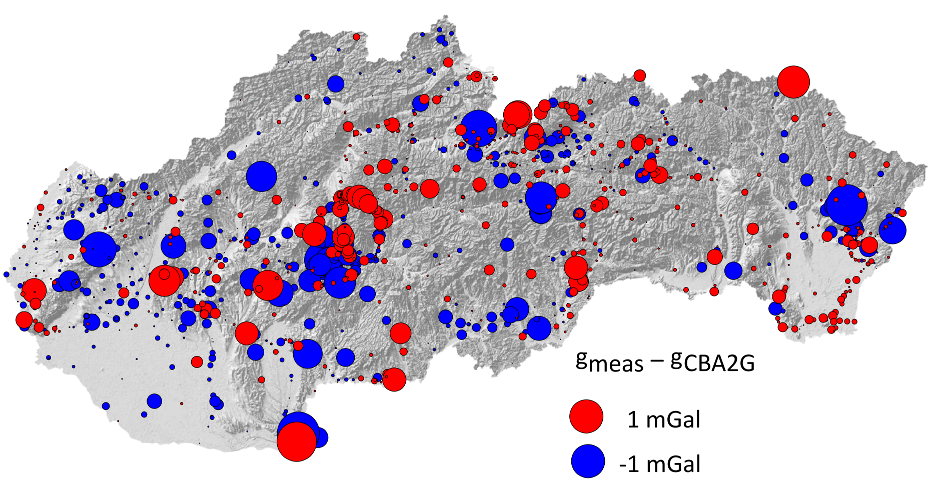

Fig. 5. Differences between the directly measured gravity and the gravity calculated by the CBA2G_SK software. Size of the circles represents the scale of differences.

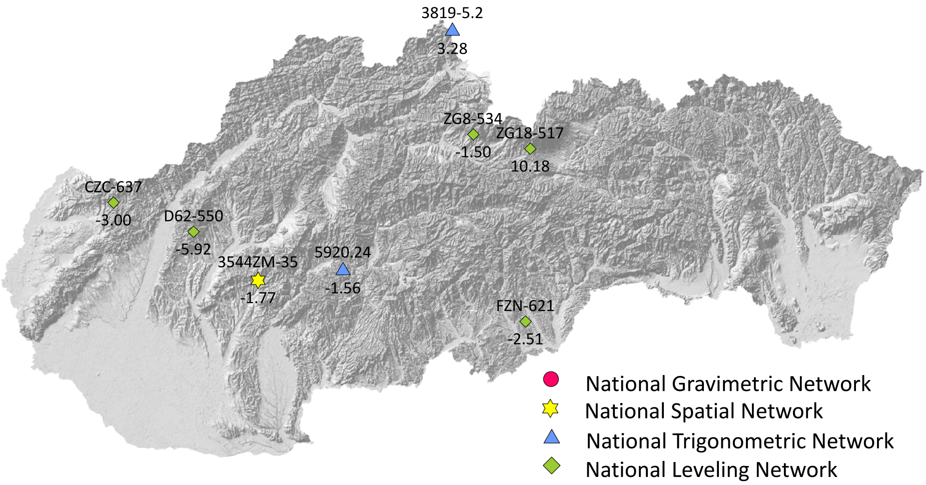

Fig. 6. Points with differences greater than ±1.5 mGal between the directly measured gravity and the gravity calculated by the CBA2G_SK software.

monic synthesis was performed with the help of the GrafLab software (*Bucha and Janák, 2014*) and the gravitational effect of the RTM was calculated using the Toposk software (*Marušiak et al., 2013*). The Toposk software is designed for the computation of terrain corrections in the zones T1–T32 (0–166.7 km) for the territory of Slovakia. Input to the Toposk contains the list of coordinates and levelling heights in the Baltic system after adjustment and DEM grids. The output file includes not only the terrain corrections but also the gravitational effect of the topographic masses (Near Topographic Effect – NTE) in the computing zones T1–T32.

The gravity on the test points is finally determined by:

$$g_{EGM+RTM} = g_{EGM2008} + RTM = g_{EGM2008} - NTE^{DTM} + NTE^{DEM}, \quad (9)$$

where NTE^{DTM} is a total gravitational effect of the topographic masses (0–166.7 km) calculated from the DTM2006.0 (*Pavlis et al., 2007*), NTE^{DEM} is a total gravitational effect of the topographic masses calculated from the detailed DEMs: zone T1: 0–250 m – DMR3 (*TOPÚ, 2012*), zone T2: 250-5240 m – DMR3-30 (*TOPÚ, 2012*); zone T31: 5240-28800 m – SRTM-3 (*Reuter et al., 2007*), zone T32: 28800–166735 m – SRTM-30 (*Reuter et al., 2007*).

When we compared the gravity from the EGM2008 (without the effect of RTM) with the directly measured gravity, the differences were in the interval from −80 to 60 mGal, with the largest differences occurring in the mountainous areas (Figure 7). The accuracy of the gravity determination depends on the accuracy of the EGM2008 model and on the quality of the point location.

In the next step we used the local RTM in combination with the EGM2008. The differences varied between −24 to 16 mGal and they are depicted in the map (Figure 8). In this case, the accuracy of the results depended not only on the accuracy of the using GGM model and the quality of the point location, but also on the quality of the used digital elevation model, i.e. DMR-3.

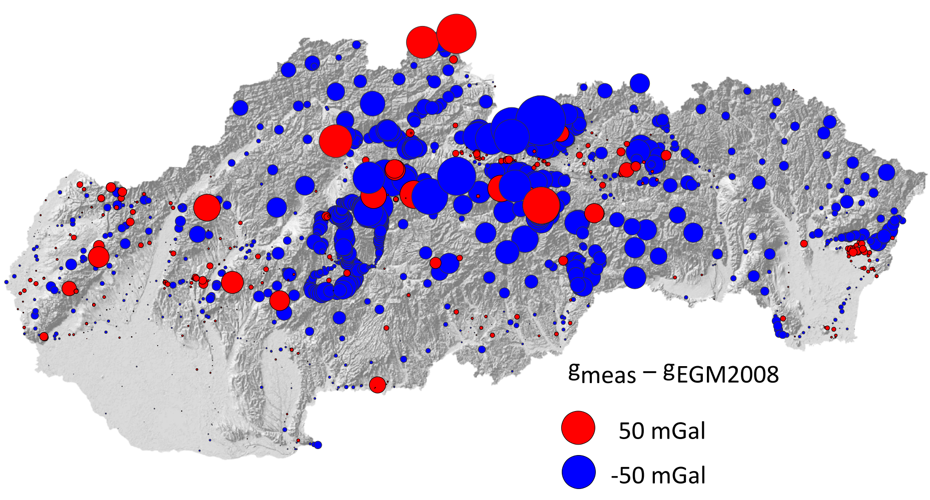

Fig. 7. Differences between the directly measured gravity and the gravity calculated from the EGM2008. Size of the circles represents the scale of differences.

3.2.3. Analyses of the gravity determination

The elementary statistic of the previously mentioned differences is shown in the next histograms (Figure 9) and Table 1. The differences ($g_{meas} - g_{CBA2G}$) are in the interval from −5.92 mGal to 10.18 mGal, but most of the differences are in the interval ±1.5 mGal and they have a low dispersion around the mean value. This analysis indicates a good quality of the

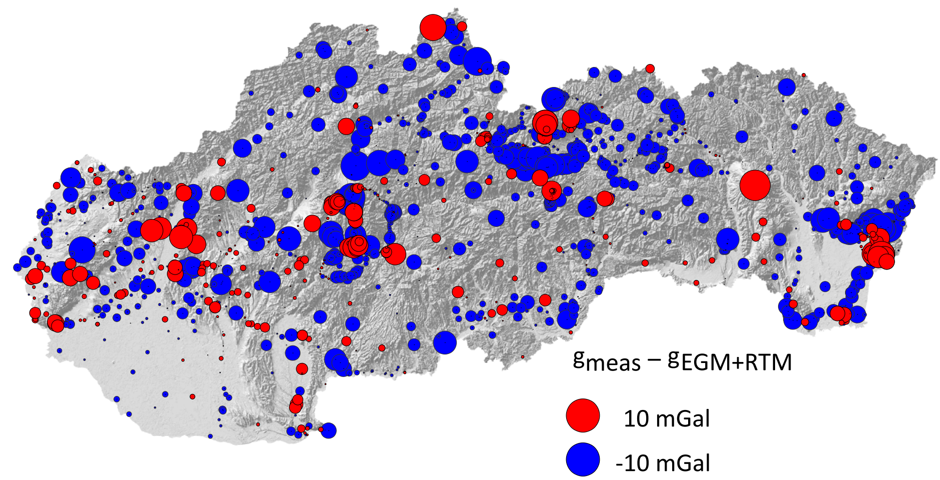

Fig. 8. Differences between the directly measured gravity and the gravity calculated from the EGM2008 and RTM. Size of the circles represents the scale of differences.

complete Bouguer anomaly map, the standard deviation of the differences is 0.46 mGal. The histogram in the middle ($g_{meas} - g_{EGM2008}$) shows that most of the differences are in the interval from -50 to 20 mGal and the mean value is systematically shifted to the value -8.15 mGal. This systematic error is probably caused by non-uniform distribution of testing points, as most of them lie along valleys. The histogram on the right presents the

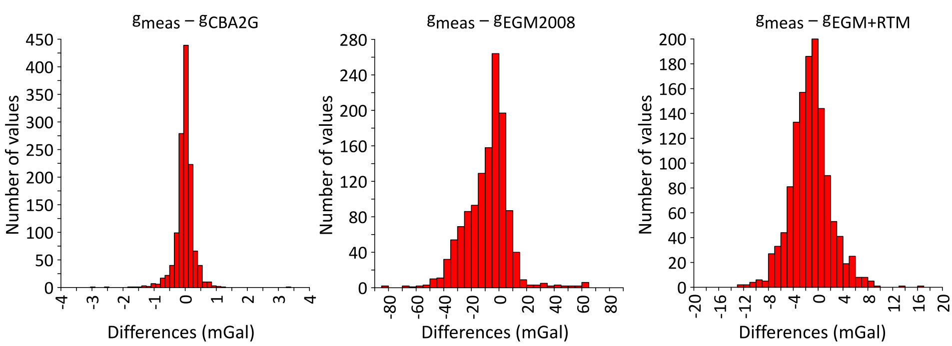

Fig. 9. Histograms of the gravity differences between the directly measured gravity and the gravity calculated by methods CBA2G_SK, EGM2008 and EGM+RTM.

Table 1. Table of the gravity differences between the directly measured gravity and the gravity calculated by methods CBA2G_SK, EGM2008 and EGM+RTM.

Number of values		**1277**		
Method		**CBA2G_SK**	**EGM2008**	**EGM+RTM**
Minimum	**mGal**	−5.924	−81.066	−23.723
Maximum		10.183	64.681	16.119
Mean		−0.012	−8.147	−1.224
Standard deviation		0.455	15.891	3.249

differences $g_{meas} - g_{EGM+RTM}$. The mean value is systematically shifted to the left (value by approx. −1.2 mGal), but most of the values are in the interval of ±8 mGal. These results clearly prove the improvement resulting from using the RTM. The standard deviation is 3.25 mGal.

3.3. Determination of the geopotential numbers

The calculated values for gravity are subsequently used to calculate geopotential numbers and normal heights. One of the reference levelling point EH-V. Pitelová has been chosen as the reference point for the area A – Pitelová. It also represents the EVRF2007 datum point No. UELN-1905325 (*Sacher et al., 2008*). The point of the 1st order in the NLN FI-782 has been chosen for the area B – Kráľova hoľa. The geopotential number in the EVRF2007 was also determined (UELN-1905337).

The process of the calculation of the geopotential numbers and normal heights is depicted step by step in the scheme in Figure 10.

Overall there are four types of heights which were based on precise levelling in combination with different ways of estimating the gravity values:

- H_{GEOP} – directly measured gravity;
- H_{CBA2G} – gravity estimated from the gravity map by the CBA2G_SK software;
- $H_{EGM2008}$ – gravity coming from the EGM2008;
- $H_{EGM+RTM}$ – gravity coming from the EGM2008 with the gravitational effect of the RTM.

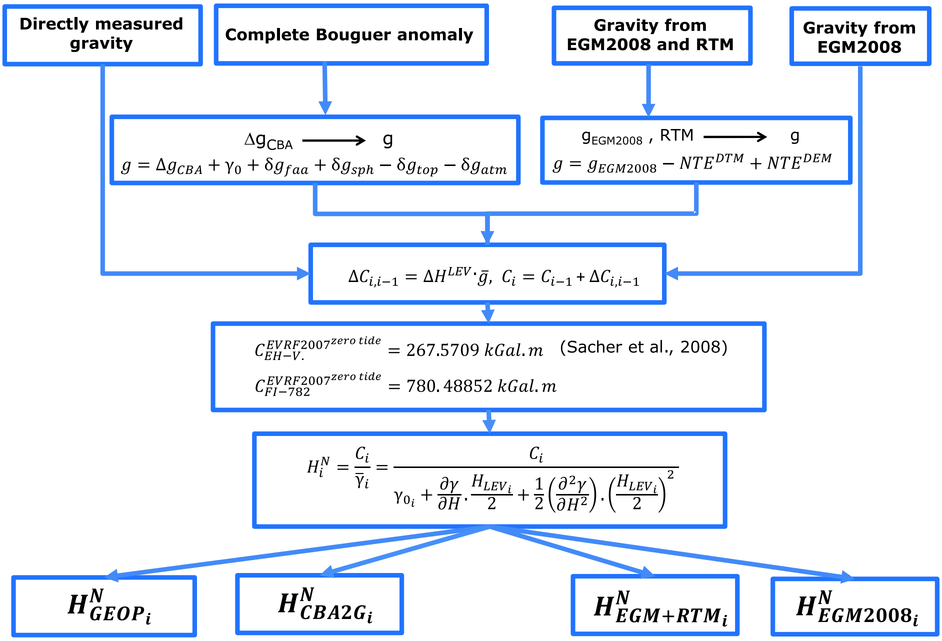

Fig. 10. Scheme of the normal heights calculation.

The graphs in Figure 11 demonstrate the differences between the normal heights computed using the directly measured gravity (H_{GEOP}) and the other alternative options for the gravity estimation (H_{CBA2G}, $H_{EGM2008}$, $H_{EGM+RTM}$). In all the graphs there are the greatest differences for the method with the gravity coming from the EGM2008 model – max. 14 mm (levelling loop of Banská Bystrica). It seems that the differences are correlated with the terrain profile. Significantly improved results are obtained after incorporating the gravitational effect of the RTM. The differences reach 3 mm at maximum in the levelling loop of Banská Bystrica. The best results in comparison with H_{GEOP} are obtained using the gravity calculated from the new map of the complete Bouguer anomaly by the CBA2G_SK software, max. 0.4 mm in the levelling line of Kráľova hoľa (Figure 13).

The levelling loop of *Banská Bystrica* is the longest one (116.6 km) and it has the biggest height range in the levelling area A – Pitelová (from 262 m to 886 m). The differences between H_{GEOP} and H_{CBA2G} are here relatively small, they vary from −0.20 to 0.00 mm (see Figures 11 and 12). The differences H_{GEOP} vs. $H_{EGM+RTM}$ are in the range of −2.73 mm to 0.02 mm. Using gravity from the EGM2008 without RTM leads to the biggest differences ranging from −5.81 to 13.45 mm (Figure 11). On some of the levelling points these differences were even bigger than the differences H_{LEV} vs. H_{GEOP}.

The levelling loop of *Handlová* has heights from 252 to 785 m and its length is 75 km. The differences H_{GEOP} vs. H_{CBA2G} are close to zero, they do not exceed 0.10 mm (Figures 11 and 12). The differences H_{GEOP} vs. $H_{EGM+RTM}$ vary from −0.47 to 2.01 mm and H_{GEOP} vs. $H_{EGM2008}$ (without gravitational effect of the RTM) range from −4.88 to 6.45 mm (Figure 11).

The levelling loop of *Banská Štiavnica* is apppox. 80 km long and its height profile ranges from 218 m to 766 m. It is evident from Figure 11 that all differences follow the height profile. Differences H_{GEOP} vs. $H_{EGM2008}$ are the biggest ones (−0.50 – 10.70 mm). By using the gravitational effect of the RTM ($H_{EGM+RTM}$) the differences were reduced to the range of −0.62 – 1.06 mm. When the gravity from the CBA2G_SK software is used, the minimum differences are again obtained (H_{GEOP} vs. H_{CBA2G}), max. 0.20 mm (Figure 12).

The short levelling line of *Pitelová* is 25.5 km long and the elevation difference between the lowest and the highest point is only 230 m. Also the height differences for all three approaches were the smallest: H_{CBA2G} minus H_{GEOP} were from −0.06 to 0 mm, $H_{EGM+RTM}$ minus H_{GEOP} from 0.16 to 0.53 mm and $H_{EGM2008}$ minus H_{GEOP} from −0.69 to 4.29 mm (Figure 12).

For the levelling line of *Kráľova hoľa* very good results were obtained for gravity values calculated by the CBA2G_SK software from the existing map of complete Bouguer anomalies (H_{CBA2G}). The differences in comparison with H_{GEOP} were in the interval of 0.00 – 0.42 mm (Figure 13 below), for $H_{EGM+RTM}$ vs. H_{GEOP} it was −0.01 to 1.05 mm and for the approach of $H_{EGM2008}$ the biggest differences were obtained: from −12.78 to 10.08 mm (Figure 13 in the middle).

In addition the gravity correction as such has been studied in the tested levelling lines, which means that the differences between raw levelling heights H_{LEV} and heights H_{GEOP} were calculated. The gravity correction depends

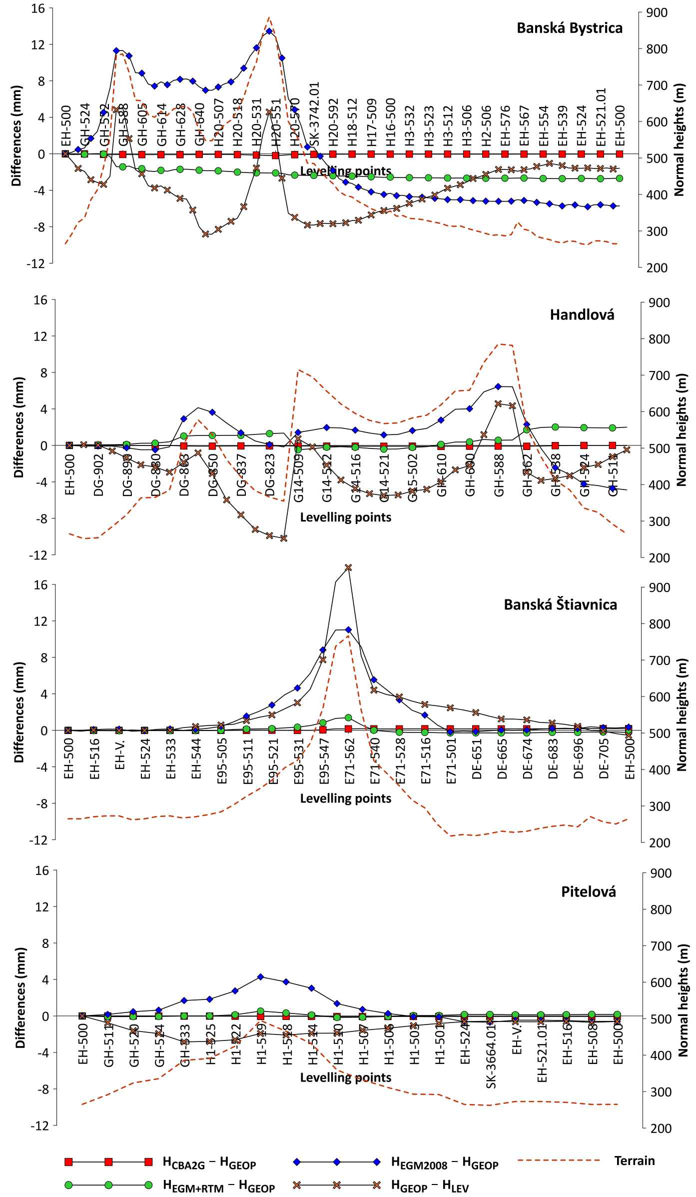

Fig. 11. Differences between the normal heights determined from the directly measured gravity (H_{GEOP}) and the normal heights determined from the gravity values calculated by the CBA2G_SK (H_{CBA2G}), from the EGM2008 ($H_{EGM2008}$), the EGM2008+RTM ($H_{EGM+RTM}$) and levelled heights (H_{LEV}) in the levelling loops of the area A – Pitelová.

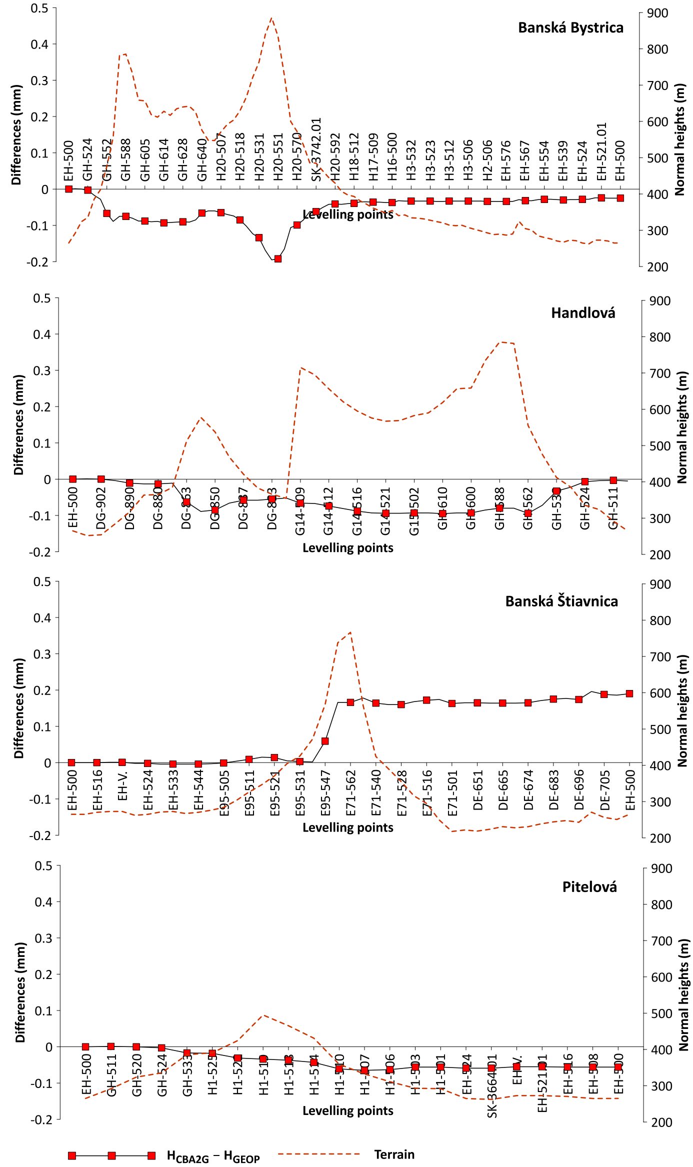

Fig. 12. Differences between the normal heights determined from the directly measured gravity (H_{GEOP}) and the normal heights determined from the gravity values calculated by the CBA2G_SK software (H_{CBA2G}) in levelling loops of the area A – Pitelová with different vertical scale compared to that of Figure 11.

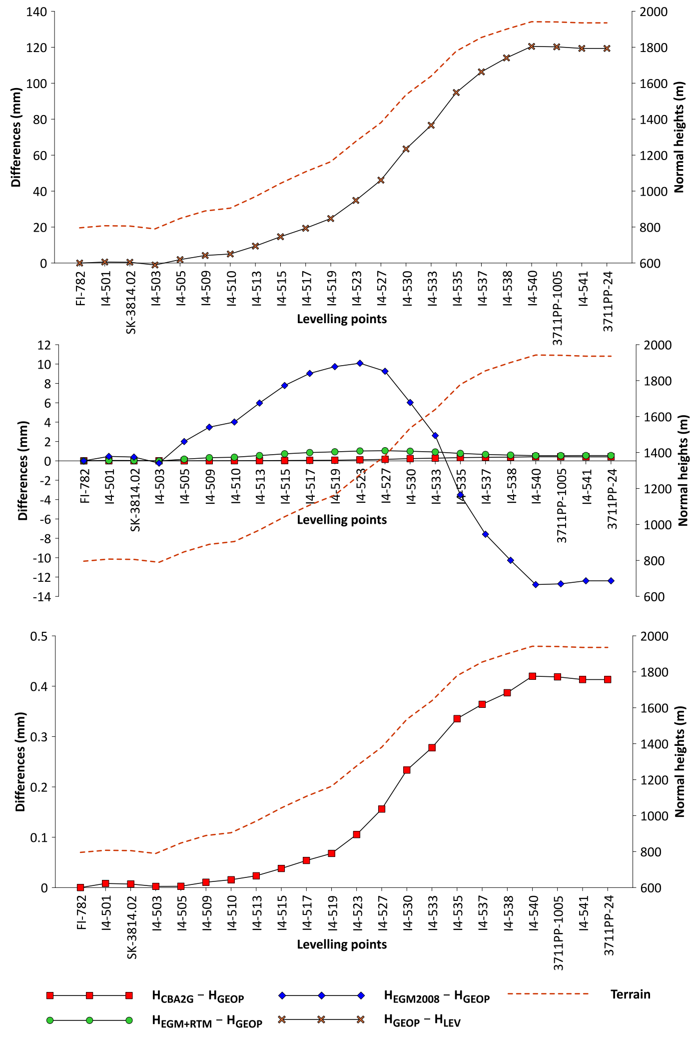

Fig. 13. Differences between the normal heights determined from the directly measured gravity (H_{GEOP}) and the normal heights determined from the gravity values calculated by the CBA2G_SK software (H_{CBA2G}), from the EGM2008 ($H_{EGM2008}$), the EGM2008+RTM ($H_{EGM+RTM}$) and levelled heights (H_{LEV}) in the levelling line of the area B – Kráľova hoľa.

Table 2. Minimum and maximum values of the normal heights and the related differences regarding various approaches to their determination.

	H_{GEOP} (m)	$H_{GEOP} - H_{LEV}$ (mm)	$H - H_{GEOP}$ (mm)		
			CBA2G_SK	**EGM2008**	**EGM+RTM**
BANSKÁ BYSTRICA					
min	262.15557	−8.82	−0.20	−5.81	−2.73
max	885.85369	5.06	0.00	13.45	0.02
mean	445.91756	−3.65	−0.06	0.63	−2.14
st. dev.	–	3.08	0.04	6.42	0.76
HANDLOVÁ					
min	251.57998	−10.19	−0.10	−4.88	−0.47
max	785.15996	4.57	0.00	6.45	2.02
mean	488.80960	−2.82	−0.05	0.92	0.64
st. dev.	–	3.17	0.04	2.75	0.82
BANSKÁ ŠTIAVNICA					
min	217.54106	−0.54	0.00	−0.16	−0.29
max	766.19052	17.85	0.20	11.03	1.39
mean	320.32580	2.24	0.09	1.84	−0.02
st. dev.	–	3.73	0.08	3.03	0.38
PITELOVÁ					
min	262.12481	−2.84	−0.07	−0.69	−0.16
max	494.81499	0.00	0.00	4.29	0.53
mean	330.35902	−1.34	−0.04	0.77	0.08
st. dev.	–	0.80	0.02	1.47	0.15
KRÁĽOVA HOĽA					
min	789.69162	−0.10	0.00	−12.78	−0.01
max	1942.10899	120.97	0.42	10.08	1.05
mean	1329.30030	50.56	0.17	−0.05	0.55
st. dev.	–	49.01	0.17	8.00	0.34

mainly on the height profile of the levelling line and the geological structure of the area in question. The results were as follows: values of max. 18 mm were found in the area A – Pitelová (levelling loop of Banská Štiavnica, see Figure 11), but in the locality B – Kráľova hoľa the gravity correction was up to 120 mm and it correlates well with the terrain profile (see upper part of Figure 13). Here the height difference between the first and the last levelling points was approximately 1150 m.

4. Conclusions

The goal of this study was to test alternative methods of gravity determination and their application for calculation of the geopotential numbers and normal heights according to Molodensky. We have tested three approaches to the gravity estimation, namely the calculation by the CBA2G_SK software using the interpolated value of the complete Bouguer anomaly, the calculation using the EGM2008 and the EGM2008 corrected by the gravitational effect of the Residual Terrain Model (RTM).

The set of testing points consisted of geodetic control points (Figure 2) with directly measured gravity. This group of points also included points with measured gravity of two levelling areas A – Pitelová and B – Kráľova hoľa.

The approach based on the CBA2G_SK software showed the minimum differences in comparison with the directly measured gravity. The majority of the differences were in the range of −1.5 mGal and +1.5 mGal (Figure 5). This indicates the good quality of the Bouguer anomaly map. In the practical experiment it was shown that gravity values determined by this approach are suitable for the calculation of the geopotential numbers and normal heights (H_{CBA2G}). The differences with respect to the heights computed using the directly measured gravity were in the interval ±0.5 mm. The accuracy of such gravity determination depends on the accuracy of the original gravimetric measurements and their processing (especially on the calculation of terrain corrections), but above all, it depends on the accuracy of the point location. The position of levelling points in the Slovak National Levelling Network is mostly interpolated from the topographic maps with the accuracy of about ±15 to 45 m (*Bublavý and Droščák, 2015*). But now the interpolation is based on the levelling point coordinates which have been obtained from different sources with centimetre-to-decimetre accuracy, but not worse than 2 m. The levelling elevations corrected by the gravity correction form the ground for calculation of the normal heights.

Two other approaches to the gravity determination were using the EGM2008 model including the gravitational effect of the RTM and the EGM2008 model without the gravitational effect of the RTM. For these approaches, in comparison with the directly measured gravity, we obtained differences in the intervals of −25 mGal to +16 mGal (EGM2008 + RTM) and of −80 to 60 mGal (EGM2008). In the method which used solely the EGM2008 model, there is an obvious correlation with the terrain (see the maps in Figures 7 and 8). Greater differences occur in more rugged terrain.

Normal heights determined using the gravity from the EGM2008 and RTM were also compared with the heights obtained from the directly measured gravity. The maximum differences of approximately 3 mm for the EGM2008 with the RTM and approximately 14 mm for the EGM2008 without the RTM were obtained. The accuracy depends on the point location, the accuracy of the EGM2008 and RTM, as well as on the accuracy of the used DEMs.

In the Slovak National Levelling Network it is necessary to guarantee the reference normal heights with sub-millimetre accuracy and this can be achieved only with the combination of the directly measured gravity and precise levelling measurements. However, when the measured gravity on all levelling points is not available, then it is necessary to use the most reliable method for the determination of gravity. This article demonstrates the fact that the new grid of the complete Bouguer anomaly in the territory of Slovakia is of sufficiently high quality (currently available with the grid cell of 100 × 100 m) and it is in principle suitable for the purpose of the calculating of the normal heights according to Molodensky in the Slovak National Levelling Network.

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